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Chapter 9

SKIN AND ON – LOAD SATURATION EFFECTS

9.1. INTRODUCTION

So far we have considered that resistances, leakage and magnetization inductances are invariable with load.

In reality, the magnetization current I_m varies only slightly from no-load to full load (from zero slip to rated slip $S_n \approx 0.01 - 0.06$), so the magnetization inductance L_{lm} varies little in such conditions.

However, as the slip increases toward standstill, the stator current increases up to (5.5 – 6.5) times rated current at stall ($S = 1$).

In the same time, as the slip increases, even with constant resistances and leakage inductances, the magnetization current I_m decreases.

So the magnetization current decreases while the stator current increases when the slip increases (Figure 9.1).

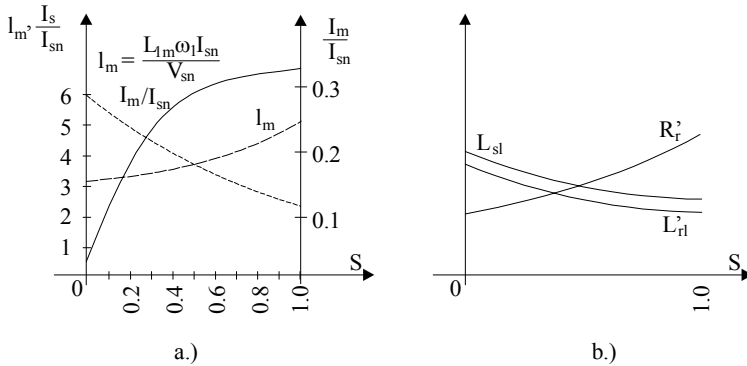


Figure 9.1 Stator I_s/I_{sn} and magnetization I_m current, magnetization inductance (L_m) in p.u. a.), leakage inductance and rotor resistance versus slip b.)

When the rotor (stator) current increases with slip, the leakage magnetic field path in iron tends to saturate. With open slots on stator, this phenomenon is limited, but, with semiopen or semiclosed slots, the slot leakage flux path saturates the tooth tops both in the stator and rotor (Figure 9.2) above (2–3) times rated current.

Also, the differential leakage inductance which is related to main flux path is affected by the tooth top saturation caused by the circumferential flux produced by slot leakage flux lines (Figure 9.2). As the space harmonics flux paths are contained within τ/π from the airgap, only the teeth saturation affects them.

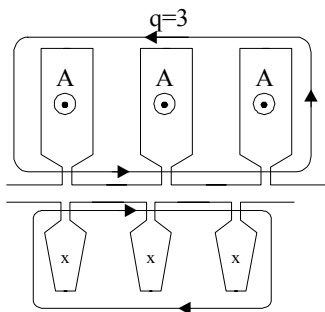


Figure 9.2 Slot leakage flux paths

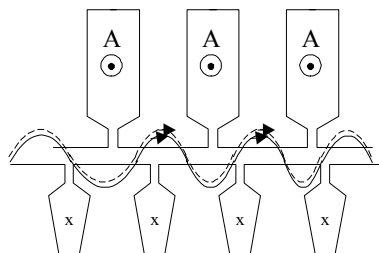


Figure 9.3 Zig-zag flux lines

Further on, for large values of stator (and rotor) currents, the zig-zag flux becomes important and contributes notably to teeth top magnetic saturation in addition to slot leakage flux contribution.

Rotor slot skewing is also known to produce variable main flux path saturation along the stack length together with the magnetization current. However the flux densities from the two contributions are phase shifted by an angle which varies and increases towards 90° at standstill. The skewing contribution to the main flux path saturation increases with slip and dominates the picture for $S > S_k$ as the magnetization flux density, in fact, decreases with slip so that at standstill it is usually 55 to 65% of its rated value.

A few remarks are in order.

- The magnetization saturation level in the core decreases with slip, such that at standstill only 55 – 65% of rated airgap flux remains.
- The slot leakage flux tends to increase with slip (current) and saturates the tooth top unless the slots are open.
- Zig – zag circumferential flux and skewing accentuate the magnetic saturation of teeth top and of entire main flux path, respectively, for high currents (above 2 to 3 times rated current).
- The differential leakage inductance is also reduced when stator (and rotor) current increases as slot, zig-zag, and skewing leakage flux effects increase.
- As the stator (rotor) current increases the main (magnetising) inductance and leakage inductances are simultaneously influenced by saturation. So leakage and main path saturation are not independent of each other. This is why we use the term: on-load saturation.

As expected, accounting for these complex phenomena simultaneously is not an easy tractable mathematical endeavour. Finite element or even refined analytical methods may be suitable. Such methods are presented in this chapter after more crude approximations ready for preliminary design are given.

Besides magnetic saturation, skin (frequency) effect influences both the resistances and slot leakage inductances. Again, a simultaneous treatment of both aspects may be practically done only through FEM.

On the other hand, if slot leakage saturation occurs only on the teeth top and the teeth, additional saturation due to skewing does not influence the flux lines distribution within the slot, the two phenomena can be treated separately.

Experience shows that such an approximation is feasible. Skin effect is treated separately for the slot body occupied by a conductor. Its influence on equivalent resistance and slot body leakage geometrical permeance is accounted for by two correction coefficients, K_R and K_X . The slot neck geometry is corrected for leakage saturation.

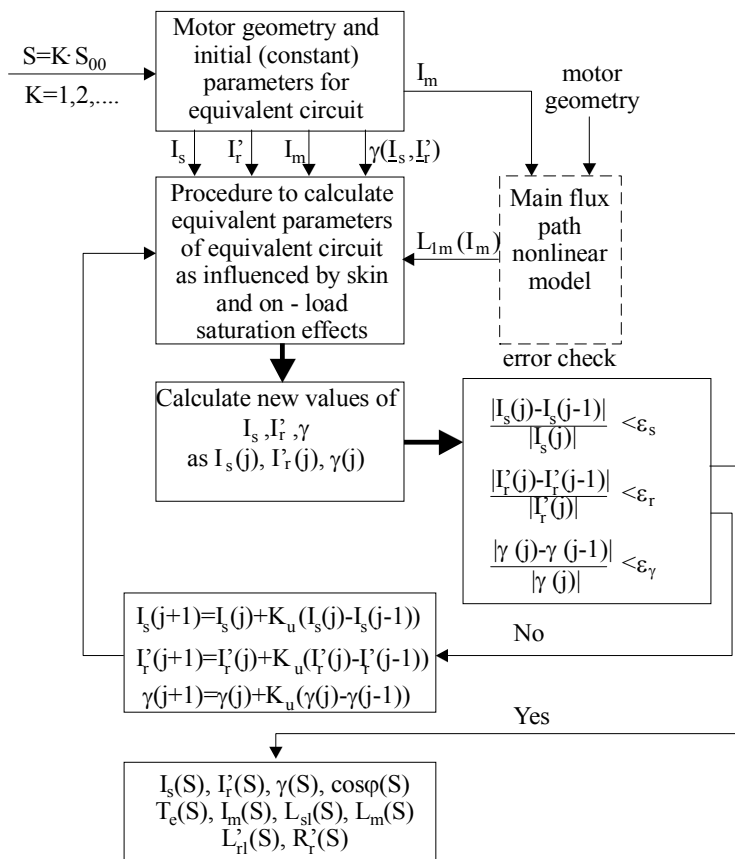


Figure 9.4 Iterative algorithm to calculate IM performance and parameters as influenced by skin and on-load saturation effects.

Finally, the on load saturation effects are treated iteratively for given slip values to find, from the equivalent circuit with variable parameters, the steady state performance. The above approach may be summarized as in [Figure 9.4](#).

The procedure starts with the equivalent circuit with constant parameters and calculates initial values of stator and rotor currents I_s, I_r' and their phase

shift angle γ . Now that we described the whole picture, let us return to its different facets and start with skin effect.

9.2. THE SKIN EFFECT

As already mentioned, skin effects are related to the flux and current density distribution in a conductor (or a group of conductors) flowed by a.c. currents and surrounded by a magnetic core with some airgaps.

Easy to use analytical solutions have been found essentially only for rectangular slots, but adaptation for related shapes has also become traditional.

More general slots with notable skin effect (of general shape) have been so far treated through equivalent multiple circuits after slicing the conductor(s) in slots in a few elements.

A refined slicing of conductor into many sections may be solved only numerically, but within a short computation time. Finally, FEM may also be used to account for skin effect. First, we will summarize some standard results for rectangular slots.

9.2.1. Single conductor in rectangular slot

Rectangular slots are typical for the stator of large IMs and for wound rotors of the same motors. Trapezoidal (and rounded) slots are typical for low power motors.

The case of a single conductor in slot is (Figure 9.5) typical to single (standard) cage rotors and is commonplace in the literature. The main results are given here.

The correction coefficients for resistance and slot leakage inductance K_R and K_X are

$$K_R = \xi \frac{(\sinh 2\xi + \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} = \frac{R_{ac}}{R_{dc}}; \quad K_X = \frac{3}{2\xi} \frac{(\sinh 2\xi - \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} = \frac{(L_{sls})_{ac}}{(L_{sls})_{dc}} \quad (9.1)$$

with

$$\xi = \beta h_s = \frac{h_s}{\delta_{Al}}; \quad \beta = \frac{1}{\delta_{Al}} = \sqrt{\frac{S\omega_1 \mu_0 \sigma_{Al}}{2}} \frac{b_c}{b_s}; \quad \sigma_{Al} - \text{electrical conductivity} \quad (9.2)$$

The slip S signifies that in this case the rotor (or secondary) of the IM is considered.

Figure 9.5 depicts K_R and K_X as functions of ξ , which, in fact, represents the ratio between the conductor height and the field penetration depth δ_{Al} in the conductor for given frequency $S\omega_1$. With one conductor in the slot, the skin effects, as reflected in K_R and K_X , increase with the slot (conductor) height, h_s , for given slip frequency $S\omega_1$.

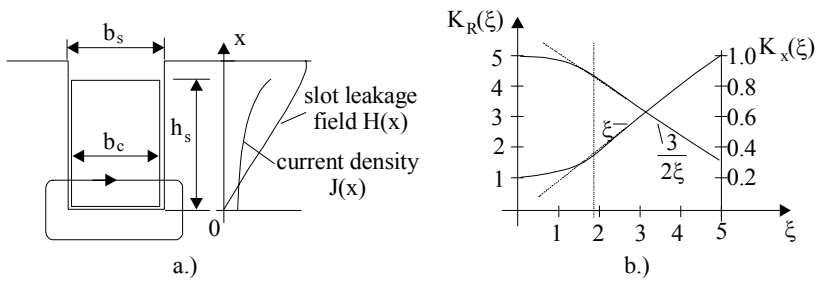


Figure 9.5 Rectangular slot
a.) slot field ($H(x)$) and current density ($J(x)$) distributions
b.) resistance K_R and slot leakage inductance K_x skin effect correction factors

This rotor resistance increase, accompanied by slot leakage inductance (reactance) decrease, leads to both a lower starting current and a higher starting torque.

This is how the deep bar cage rotor has evolved. To increase further the skin effects, and thus increase starting torque for even lower starting current ($I_{\text{start}} = (4.5-5)I_{\text{rated}}$), the double cage rotor was introduced by the turn of this century already by Dolivo – Dobrovolski and later by Boucherot.

The advent of power electronics, however, has led to low frequency starts and thus, up to peak torque at start, may be obtained with (2.5–3) times rated current. Skin effect in this case is not needed. Reducing skin effect in large induction motors with cage rotors lead to particular slot shapes adequate for variable frequency supply.

9.2.2. Multiple conductors in rectangular slots: series connection

Multiple conductors are placed in the stator slots, or in the rotor slots of wound rotors (Figure 9.6).

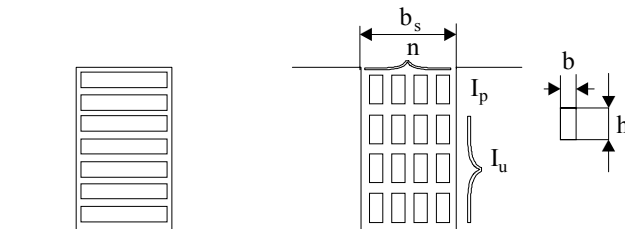


Figure 9.6 Multiple conductors in rectangular slots

According to Emde and R.Richter [1,2] who continued the classic work of Field [3], the resistance correction coefficient K_{Rp} for the p^{th} layer in slot (Figure 9.6) with current I_p , when total current below p^{th} layer is I_u , is

$$K_{RP} = \varphi(\xi) + \frac{I_u(I_u \cos \gamma + I_p)}{I_p^2} \psi(\xi) \quad (9.3)$$

$$\varphi(\xi) = \xi \frac{(\sinh 2\xi + \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)}; \quad \psi(\xi) = 2\xi \frac{(\sinh \xi - \sin \xi)}{(\cosh \xi + \cos \xi)} \quad (9.4)$$

$$\xi = \beta_n h; \quad \beta_n = \sqrt{\frac{S\omega_1 \mu_0 \sigma_{Al}}{2} \frac{nb}{b_s}}$$

There are n conductors in each layer and γ is the angle between I_p and I_u phasors.

In two-layer windings with chorded coils, there are slots where the current in all conductors is the same and some in which two phases are located and thus the currents are different (or there is a phase shift $\gamma = 60^\circ$).

For the case of $\gamma = 0$ with $I_u = I_p(p - 1)$ Equation (9.3) becomes

$$K_{RP} = \varphi(\xi) + (p^2 - p)\psi(\xi) \quad (9.5)$$

This shows that the skin effect is not the same in all layers. The average value of K_{RP} for m layers,

$$K_{Rm} = \frac{1}{m} \sum_1^m K_{RP}(p) = \varphi(\xi) + \frac{m^2 - 1}{3} \psi(\xi) > 1 \quad (9.6)$$

Based on [4], for $\gamma \neq 0$ in (9.6) $(m^2 - 1)/3$ is replaced by

$$\frac{m^2(5 + 3 \cos \gamma)}{24} - \frac{1}{3} \quad (9.6')$$

A similar expression is obtained for the slot-body leakage inductance correction K_x [4].

$$K_{xm} = \varphi'(\xi) + \frac{(m^2 - 1)\psi'(\xi)}{m^2} < 1 \quad (9.7)$$

$$\varphi'(\xi) = \frac{3}{2\xi} \frac{(\sinh 2\xi - \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} \quad (9.8)$$

$$\psi'(\xi) = \frac{(\sinh \xi + \sin \xi)}{\xi(\cosh \xi + \cos \xi)} \quad (9.9)$$

Please note that the first terms in K_{Rm} and K_{xm} are identical to K_R and K_x of (9.1) valid for a single conductor in slot. As expected, K_{Rm} and K_{xm} degenerate into K_R and K_x for one layer (conductor) per slot. The helping functions φ , ψ , φ' , ψ' are quite general (Figure 9.7).

For a given slot geometry, increasing the number of conductor layers in slot reduces their height $h = h_s/m$ and thus reduces ξ , which ultimately reduces $\psi(\xi)$ in (9.6). On the other hand, increasing the number of layers, the second term in (9.6) tends to increase.

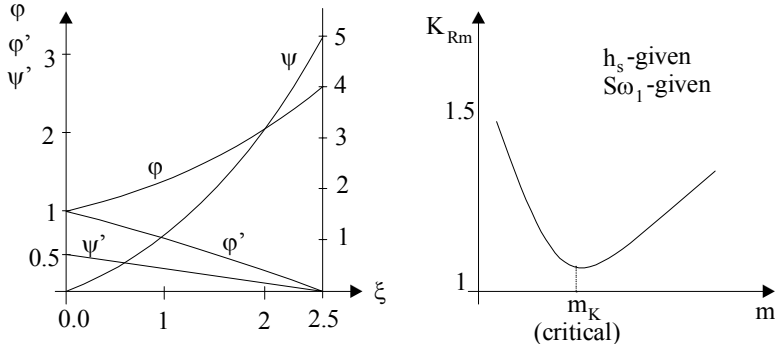


Figure 9.7 Helping functions ϕ, Ψ, ϕ', Ψ' versus ξ

It is thus evident that there is a critical conductor height h_c for which the resistance correction coefficient is minimum. Reducing the conductor height below h_c does not produce a smaller K_{Rm} .

In large power or in high speed (frequency), small/medium power machines this problem of critical conductor height is of great importance to minimize the additional (a.c.) losses in the windings.

A value of $K_{Rm} \approx (1.1 - 1.2)$ is in most cases, acceptable. At power grid frequency (50 – 60 Hz), the stator skin effect resistance correction coefficient is very small (close to 1.0) as long as power is smaller than a few hundred kW.

Inverter-fed IMs, however, show high frequency time harmonics for which K_{Rm} may be notable and has to be accounted for.

Example 9.1. Derivation of resistance and reactance corrections

Let us calculate the magnetic field $H(x)$ and current density $J(x)$ in the slot of an IM with m identical conductors (layers) in series making a single layer winding.

Solution

To solve the problem we use the field equation in complex numbers for the slot space where only along slot depth (OX) the magnetic field and current density vary.

$$\frac{\partial^2 \underline{H}(x)}{\partial x^2} = j \frac{b}{b_s} \omega \mu_0 \sigma_{Co} \underline{H}(x) \quad (9.10)$$

The solution of (9.10) is

$$\underline{H}(x) = \underline{C}_1 e^{-(1+j)\beta x} + \underline{C}_2 e^{+(1+j)\beta x}, \quad \beta = \sqrt{\frac{b}{b_s} \frac{\omega \mu_0 \sigma}{2}} \quad (9.11)$$

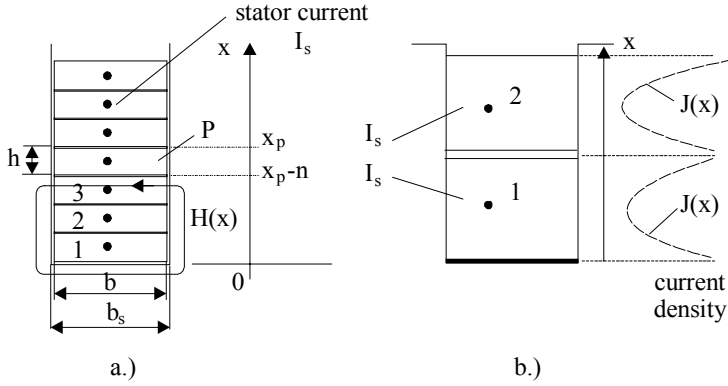


Figure 9.8 Stator slot with single coil with m layers (conductors in series) a.) and two conductors in series b.)

The boundary conditions are

$$\begin{aligned} \underline{H}(x_p) \cdot b_s &= \underline{I}_s p; \quad x = x_p; \quad x_p = ph \\ \underline{H}(x_p - h) \cdot b_s &= \underline{I}_s (p - 1); \quad x = x_p - h \end{aligned} \quad (9.12)$$

From (9.11) and (9.12), we get the expressions of the constants $\underline{C}_1$ and $\underline{C}_2$

$$\begin{aligned} \underline{C}_1 &= \frac{\underline{I}_s}{2b_s \sinh[(1+j)\beta h]} \left[(p-1)e^{(1+j)\beta x_p} - pe^{(1+j)\beta(x_p-h)} \right] \\ \underline{C}_2 &= \frac{\underline{I}_s}{2b_s \sinh[(1+j)\beta h]} \left[-(p-1)e^{-(1+j)\beta x_p} + pe^{-(1+j)\beta(x_p-h)} \right] \end{aligned} \quad (9.13)$$

The current density $\underline{J}(x)$ is

$$\underline{J}(x) = -\frac{b_s}{b} \frac{\partial \underline{H}(x)}{\partial x} = -\frac{b_s}{b} \beta (1+j) \left[\underline{C}_1 e^{-(1+j)\beta x} - \underline{C}_2 e^{(1+j)\beta x} \right] \quad (9.14)$$

For $m = 2$ conductors in series per slot, the current density distribution (9.14) is as shown qualitatively in [Figure 9.8](#).

The active and reactive powers in the p^{th} conductor $\underline{S}_p$ is calculated using the Poyting vector [4].

$$\underline{S}_{a.c.} = P_{a.c.} + jQ_{a.c.} = \frac{b_s L}{\sigma_{Co}} \left[\left(\frac{\underline{J}}{2} \frac{\underline{H}^*}{2} \right)_{x=x_p-h} - \left(\frac{\underline{J}}{2} \frac{\underline{H}^*}{2} \right)_{x=x_p} \right] \quad (9.15)$$

Denoting by R_{pa} and X_{pa} the a.c. resistance and reactance of conductor p , we may write

$$P_{ac} = R_{ac} I_s^2 \quad Q_{ac} = X_{ac} I_s^2 \quad (9.16)$$

The d.c. resistance R_{dc} and reactance X_{dc} of conductor p,

$$R_{dc} = \frac{1}{\sigma_{Co}} \frac{L}{hb}; \quad X_{dc} = \omega \mu_0 \frac{b_s L}{3h}; \quad L - \text{stack length} \quad (9.17)$$

The ratios between a.c. and d.c. parameters K_{Rp} and K_{xp} are

$$K_{Rp} = \frac{R_{ac}}{R_{dc}}; \quad K_{xp} = \frac{X_{ac}}{X_{dc}} \quad (9.18)$$

Making use of (9.11) and (9.14) leads to the expressions of K_{Rp} and K_{xp} represented by (9.5) and (9.6).

9.2.3. Multiple conductors in slot: parallel connection

Conductors are connected in parallel to handle the phase current, In such a case, besides the skin effect correction K_{Rm} , as described in paragraph 9.3.2 for series connection, circulating currents will flow between them. Additional losses are produced this way.

When multiple round conductors in parallel are used, their diameter is less than 2.5(3) mm and thus, at least for 50(60) Hz machines, the skin effect may be neglected altogether. In contrast, for medium and large power machines, with rectangular shape conductors (Figure 9.9), the skin effect influence has at least to be verified. In this case also, the circulating current influence is to be considered.

A simplified solution to this problem [5] is obtained by neglecting, for the time being, the skin effect of individual conductors (layers), that is by assuming a linear leakage flux density distribution along the slot height. Also the inter-turn insulation thickness is neglected.

At the junction between elementary conductors (strands), the average a.c. magnetic flux density $B_{ave} \approx B_m/4$ (Figure 9.11a). The a.c. flux through the cross section of a strand Φ_{ac} is

$$\Phi_{ac} = B_{ave} h l_{stack} \quad (9.19)$$

The d.c. resistance of a strand R_{dc} is

$$R_{ac} \approx R_{dc} = \frac{1}{\sigma_{Co}} \frac{l_{turn}}{bh} \quad (9.20)$$

Now the voltage induced in a strand turn E_{ac} is

$$E_{ac} = \omega \Phi_{ac} \quad (9.21)$$

So the current in a strand I_{st} , with the leakage inductance of the strand neglected, is:

$$I_{st} = E_{ac} / R_{ac} \quad (9.22)$$

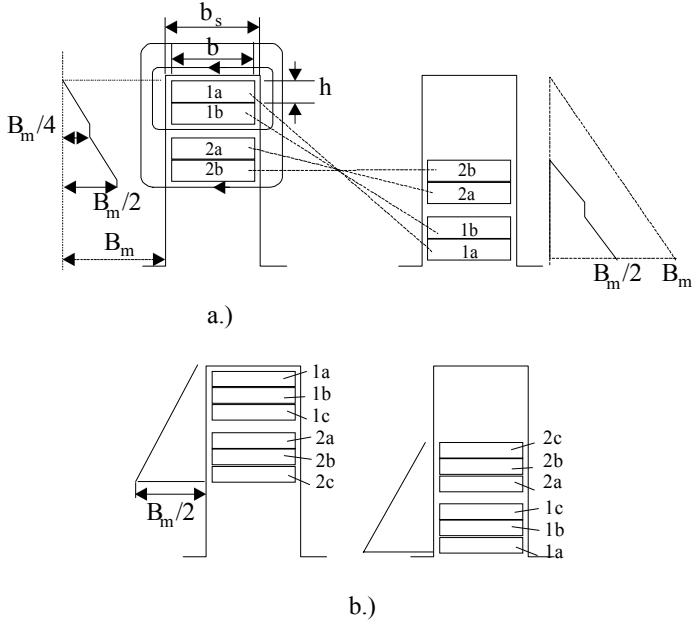


Figure 9.9 Slot leakage flux density for coil sides: two turn coils
a.) two elementary conductors in parallel (strands) b.) three elementary conductors in parallel

The loss in a strand P_{strand} is

$$P_{\text{strand}} = \frac{E_{\text{ac}}^2}{R_{\text{ac}}} = \frac{\omega^2 B_{\text{ave}}^2 h^2 l_{\text{stack}}^2}{\frac{1}{\sigma_{\text{Co}}} \frac{l_{\text{turn}}}{bh}} \quad (9.23)$$

As seen from Figure 9.9a, the average flux density B_{ave} is

$$B_{\text{ave}} = \frac{B_m}{4} = \frac{\mu_0 n_{\text{coil}} I_{\text{phase}} (1 + \cos \gamma)}{4b_s} \quad (9.24)$$

I_{phase} is the phase current and γ is the angle between the currents in the upper and lower coils. Also, n_{coil} is the number of turns per coil (in our case $n_{\text{coil}} = 2,3$).

The usual d.c. loss in a strand with current (two vertical strands / coil) is

$$P_{\text{dc}} = R_{\text{dc}} \left(\frac{I_{\text{phase}}}{2} \right)^2 \quad (9.25)$$

We may translate the circulating new effect into a resistance additional coefficient, K_{Rad} .

$$K_{\text{Rad}} \approx \frac{P_{\text{strand}}}{P_{\text{dc}}} = \omega^2 \mu_0^2 \sigma_{\text{Co}} \frac{b^2 h^4}{b_s^2} \left(\frac{l_{\text{stack}}}{l_{\text{turn}}} \right)^2 \frac{n_{\text{coil}}^2 (1 + \cos \gamma)^2}{4} \quad (9.26)$$

Expression (9.26) is strictly valid for two vertical strands in parallel. However as B_{ave} seems to be the same for other number of strands/turn, Equation (9.26) should be valid in general.

Adding the skin effect coefficient K_{Rm} as already defined to the one due to circulating current between elementary conductors in parallel, we get the total skin effect coefficient $K_{\text{R||}}$.

$$K_{\text{R||}} = K_{\text{Rm}} \frac{l_{\text{stack}}}{l_{\text{turn}}} + K_{\text{Rad}} \quad (9.26')$$

Even with large power IMs, $K_{\text{R||}}$ should be less than 1.25 to 1.3 with $K_{\text{Rad}} < 0.1$ for a proper design.

Example 9.2. Skin effect in multiple vertical conductors in slot

Let us consider a rather large induction motor with 2 coils, each made of 4 elementary conductors in series, respectively, and, of two turns, each of them made of two vertical strands (conductors in parallel) per slot in the stator. The size of the elementary conductor is $h \cdot b = 5.20$ [mm·mm] and the slot width $b_s = 22$ mm; the insulation thickness along slot height is neglected. The frequency $f_1 = 60$ Hz. Let us determine the skin effect in the stack zone for the two cases, if $l_{\text{stack}}/l_{\text{turn}} = 0.5$.

Solution

As the elementary conductor is the same in both cases, the first skin effect resistance correction coefficient K_{Rm} may be computed first from (9.6) with ξ from (9.4),

$$\begin{aligned} \xi &= \beta_n h; \quad h = 8 \text{ mm} \\ \beta_n &= \sqrt{\frac{\omega_1 \mu_0 \sigma_{\text{Co}}}{2} \frac{b}{b_s}} = \sqrt{\frac{2\pi 60 \cdot 1.256 \cdot 10^{-6}}{2 \cdot 1.8 \cdot 10^{-8}} \frac{20}{22}} = 109.32 \text{ m}^{-1} \\ \xi &= 109.32 \cdot 5 \cdot 10^{-3} = 0.5466 \end{aligned}$$

The helping functions $\varphi(\xi)$ and $\psi(\xi)$ are (from (9.7)): $\varphi(\xi) = 1.015$, $\psi(\xi) = 0.04$. Now with $m = 8$ layers in slot K_{Rm} (9.6) is

$$K_{\text{Rm}} = 1.015 + \frac{8^2 - 1}{3} 0.04 = 1.99$$

Now, for the parallel conductors (2 in parallel), the additional resistance correction coefficient K_{Rad} (9.26) for circulating currents is

$$K_{\text{Rad}} = (1.256 \cdot 10^{-6})^2 (2\pi 60)^2 \frac{1}{(1.8 \cdot 10^{-8})^2} \left(\frac{20}{22}\right)^2 \cdot \frac{(5 \cdot 10^{-3})^4 \cdot 0.5^2 \cdot 2^2 \cdot (1+1)^2}{4} = 0.3918!$$

The coefficient K_{Rad} refers to the whole conductor (turn) length, that is, it includes the end-turn part of it. K_{Rm} is too large, to be practical.

9.2.4. The skin effect in the end turns

There is a part of stator and rotor windings that is located outside the lamination stack, mainly in air: the end turns or endrings.

The skin effect for conductors in air is less pronounced than in their portions in slots.

As the machine power or frequency increases, this kind of skin effect is to be considered. In Reference [6] the resistance correction coefficient K_R for a single round conductor (d_{Co}) is also a function of β in the form (Figure 9.10).

$$\xi = d_{\text{Co}} \sqrt{\frac{\omega_l \mu_0 \sigma_{\text{Co}}}{2}} \quad (9.27)$$

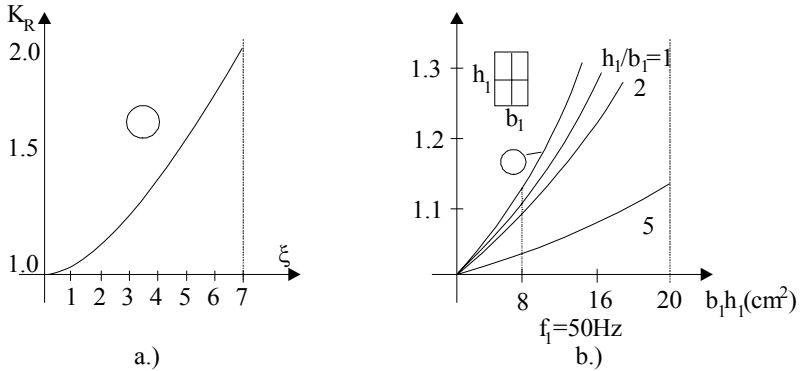


Figure 9.10 Skin effect correction factor K_R for a round conductor in air:
a.) circular b.) rectangular

On the other hand, a rectangular conductor in air [7] presents the resistance correction coefficient (Figure 9.10) based on the assumption that there are magnetic field lines that follow the conductor periphery.

In general, there are m layers of round or rectangular conductors on top of each other (Figure 9.11).

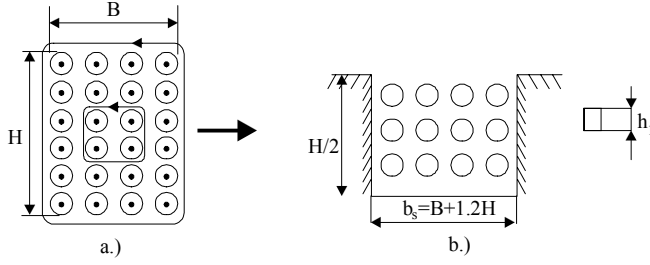


Figure 9.11 Four layer coil in air a.) and its upper part placed in an equivalent (fictious) slot

Now the value of ξ is

$$\xi = d_{Co} \sqrt{\frac{\omega_1 \mu_0 \sigma}{2} \frac{B}{B + 1.2H}} \quad \text{for round conductors} \quad (9.28)$$

$$\xi = h_1 \sqrt{\frac{\omega_1 \mu_0 \sigma}{2} \frac{B}{B + 1.2H}} \quad \text{for rectangular conductors}$$

As the skin effect is to be reduced, ξ should be made smaller than 1.0 by design. And, in this case, for rectangular conductors displaced in m layers [2], the correction coefficient K_{Rme} is

$$K_{Rme} = 1 + \frac{(m^2 - 0.8)}{36} \xi^4 \quad (9.29)$$

For a bundle of Z round conductors [24] K_{Rme} is

$$K_{Rme} = 1 + 0.005 \cdot Z \cdot (d / \text{cm})^4 (f / 50\text{Hz})^2 \quad (9.29')$$

The skin effect in the endrings of rotors may be treated as a single rectangular conductor in air. For small induction machines, however, the skin effect in the endrings may be neglected. In large IMs, a more complete solution is needed. This aspect will be treated later in this chapter.

For the IM in example 9.2, with $m = 4$, $\xi = 0.5466$, the skin effect in the end turns K_{Rme} (9.29) is

$$K_{Rme} = 1 + \frac{4^2 - 0.8}{36} 0.5466^2 = 1.0377!$$

As expected, $K_{Rme} \ll K_{Rm}$ corresponding to the conductors in slot. The total skin effect resistance correction coefficient K_{Rt} is

$$K_{Rt} = \frac{K_{Rm} l_{stack} + K_{Rme} (l_{coil} - l_{stack})}{l_{coil}} + K_{Rad} \quad (9.30)$$

For the case of example 9.2,

$$K_{Rt} = \frac{1.99 + 1.0377 \left(1 - \frac{1}{1.5}\right)}{1.5} + 0.3918 = 1.5572 + 0.3918 = 1.949$$

for 2 conductors in parallel and $K_{Rt} = 1.5572$, for all conductors in series.

9.3. SKIN EFFECTS BY THE MULTILAYER APPROACH

For slots of more general shape, adopted to exploit the beneficial effects of rotor cages, a simplified solution is obtained by dividing the rotor bar into n layers of height h_l and width b_j (Figure 9.12). The method originates in [1].

For the p^{th} layer Faraday's law yields

$$R_p I_p - R_{p+1} I_{p+1} = -j S \omega_l \Delta \Phi_p \quad (9.31)$$

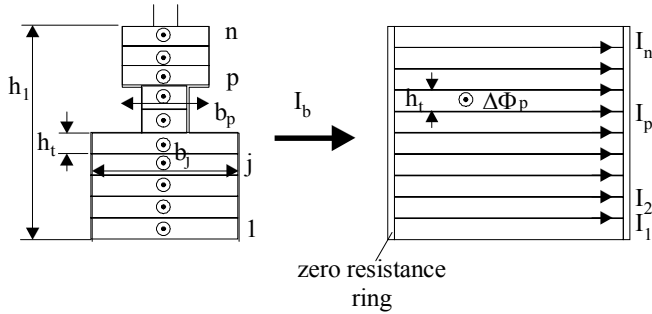


Figure 9.12 More general shape rotor bars

$$R_p = \frac{1}{\sigma_{Al}} \frac{l_{stack}}{b_p h_t}; \quad R_{p+1} = \frac{1}{\sigma_{Al}} \frac{l_{stack}}{b_{p+1} h_t}; \quad \Delta \Phi_p = \frac{\mu_0 l_{stack} h_t}{b_p} \sum_{j=1}^p I_j \quad (9.32)$$

R_p and R_{p+1} represent the resistances of p^{th} and $(p+1)^{\text{th}}$ layer and L_p the inductance of p^{th} layer.

$$L_p = \frac{\mu_0 l_{stack} h_t}{b_p} \quad (9.33)$$

With (9.33), Equation (9.31) becomes

$$I_{p+1} = \frac{R_p}{R_{p+1}} I_p + j \frac{S \omega_l L_p}{R_{p+1}} \sum_{j=1}^p I_j \quad (9.34)$$

Let us consider $p = 1, 2$ in (9.34)

$$I_2 = \frac{R_1}{R_2} I_1 + j \frac{S \omega_l L_1}{R_2} I_1 \quad (9.35)$$

$$I_3 = \frac{R_2}{R_3} I_2 + j \frac{S\omega_1 L_2}{R_3} (I_1 + I_2) \quad (9.36)$$

If we assign a value to I_1 in relation to total current I_b , say,

$$(I_1)_{\text{initial}} = \frac{I_b}{n}, \quad (9.37)$$

we may use Equations (9.34) through (9.36) to determine the current in all layers. Finally,

$$(I_b)' = \left| \sum_{j=1}^n I_j \right| \quad (9.38)$$

As expected, I_b and I_b' will be different. Consequently, the currents in all layers will be multiplied by I_b/I_b' to obtain their real values. On the other hand, Equations (9.35) – (9.36) lead to the equivalent circuit in [Figure 9.12](#).

Once the layer currents $I_1, \dots, I_n$ are known, the total losses in the bar are

$$P_{ac} = \sum_{j=1}^n |I_j|^2 R_j \quad (9.39)$$

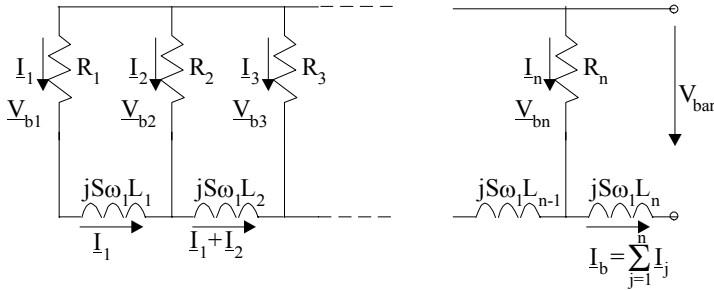


Figure 9.13 Equivalent circuit for skin effect evaluation

In a similar manner, the magnetic energy in the slot W_{mac} is

$$W_{\text{mac}} = \frac{1}{2} \sum_{j=1}^n L_j \left| \sum_{k=1}^j I_k \right|^2 \quad (9.40)$$

The d.c. power loss in the slot (for given total bar current) P_{dc} is

$$P_{dc} = \sum I_{jdc}^2 R_j; \quad I_{jdc} = \frac{I_b'}{A_{\text{bar}}} h_t b_j \quad (9.41)$$

Also the d.c. magnetic energy in the slot

$$W_{\text{mdc}} = \frac{1}{2} \sum_{j=1}^n L_j \left| \sum_1^j I_{\text{kdc}} \right|^2 \quad (9.42)$$

Now the skin effect resistance and inductance correction coefficients K_R , K_x are

$$K_R = \frac{P_{ac}}{P_{dc}} = \frac{\sum_{j=1}^n I_j^2 R_j}{\sum_{j=1}^n I_{jdc}^2 R_j} \quad (9.43)$$

$$K_x = \frac{\left| \sum_{j=1}^n L_j \left| \sum_{k=1}^j I_k \right|^2 \right|}{\left| \sum_{j=1}^n L_j \left| \sum_{k=1}^j I_{kdc} \right|^2 \right|} \quad (9.44)$$

Example 9.3. Let us consider a deep bar of the shape in Figure 9.12 with the dimensions as in Figure 9.14.

Let us divide the bar into only 6 layers, each 5 mm high ($h_t = 5$ mm) and calculate the skin effects for $S = 1$ and $f_1 = 60$ Hz.

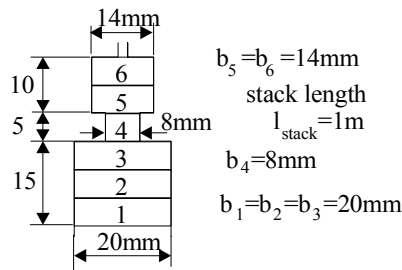


Figure 9.14 Deep bar geometry

Solution

From Figure 9.14, the layer resistances and inductances are (9.32 – 9.33).

$$R_1 = R_2 = R_3 = \frac{1}{\sigma_{Al}} \frac{l_{stack}}{b_l h_t} = \frac{1}{3 \cdot 10^7} \frac{1}{20 \cdot 10^{-3} \cdot 5 \cdot 10^{-3}} = 0.333 \cdot 10^{-3} \Omega$$

$$R_4 = \frac{1}{\sigma_{Al}} \frac{l_{stack}}{b_4 h_t} = \frac{1}{3 \cdot 10^7} \frac{1}{8 \cdot 10^{-3} \cdot 5 \cdot 10^{-3}} = 0.833 \cdot 10^{-3} \Omega$$

$$R_5 = R_6 = \frac{1}{\sigma_{Al}} \frac{l_{stack}}{b_5 h_t} = \frac{1}{3 \cdot 10^7} \frac{1}{14 \cdot 10^{-3} \cdot 5 \cdot 10^{-3}} = 0.476 \cdot 10^{-3} \Omega$$

From (9.33)

$$L_1 = L_2 = L_3 = \frac{\mu_0 l_{stack} h_t}{b_1} = 1.256 \cdot 10^{-6} \cdot 1 \cdot \frac{0.005}{0.020} = 0.314 \cdot 10^{-6} H$$

$$L_4 = \frac{\mu_0 l_{stack} h_t}{b_5} = 1.256 \cdot 10^{-6} \cdot \frac{0.005}{0.008} = 0.785 \cdot 10^{-6} H$$

$$L_5 = L_6 = \frac{\mu_0 l_{stack} h_t}{b_5} = 1.256 \cdot 10^{-6} \cdot \frac{0.005}{0.014} = 0.44857 \cdot 10^{-6} H$$

Let us now consider that the bar current is $I_b = 3600A$ and $I_1 = I_b/n = I_b/6 = 600A$. Now $\underline{I}_2$ (in the second layer from slot bottom) is

$$\underline{I}_2 = \frac{R_1}{R_2} \underline{I}_1 + j \frac{S \omega_1 L_1}{R_2} \underline{I}_1 = 600 + j \cdot 1 \cdot 2\pi 60 \cdot \frac{0.314 \cdot 10^{-6}}{0.333 \cdot 10^{-3}} \cdot 600 = 600 + j213.18A$$

$$|\underline{I}_2| = 634.74A$$

$$\begin{aligned} \underline{I}_3 &= \frac{R_2}{R_3} \underline{I}_2 + j \frac{S \omega_1 L_2}{R_3} (\underline{I}_1 + \underline{I}_2) = 600 + j213.18 + \\ &+ j \cdot 1 \cdot 2\pi 60 \cdot \frac{0.314 \cdot 10^{-6}}{0.333 \cdot 10^{-3}} \cdot (600 + 600 + j213.18) = 524.25 + j640A \end{aligned}$$

$$|\underline{I}_3| = 827.3A$$

$$\begin{aligned} \underline{I}_4 &= \frac{R_3}{R_4} \underline{I}_3 + j \frac{S \omega_1 L_3}{R_4} (\underline{I}_1 + \underline{I}_2 + \underline{I}_3) = \frac{0.333 \cdot 10^{-3}}{0.833 \cdot 10^{-3}} (524.25 + j426.26) + \\ &+ j \cdot 1 \cdot 2\pi 60 \cdot \frac{0.314 \cdot 10^{-6}}{0.833 \cdot 10^{-3}} \cdot (600 + 600 + j213.18 + 524 + j640) \\ &= 55.5 + j490.2A \end{aligned}$$

$$|\underline{I}_4| = 493.27A$$

$$\begin{aligned} I_5 &= \frac{R_4}{R_5} I_4 + j \frac{S \omega_1 L_4}{R_5} (I_1 + I_2 + I_3 + I_4) = \frac{0.833 \cdot 10^{-3}}{0.476 \cdot 10^{-3}} (55.5 + j490.2) + \\ &+ j \cdot 1 \cdot 2\pi 60 \cdot \frac{0.785 \cdot 10^{-6}}{0.4485 \cdot 10^{-3}} \cdot (1779.75 + j1342) \\ &= -712.088 + j2030.4A \end{aligned}$$

$$|I_5| = 2151.2A$$

$$\begin{aligned} I_6 &= \frac{R_5}{R_6} I_5 + j \frac{S \omega_1 L_5}{R_6} (I_1 + I_2 + I_3 + I_4 + I_5) = \\ &= \frac{0.476 \cdot 10^{-3}}{0.476 \cdot 10^{-3}} (-712.088 + j2030.4) + \\ &+ j \cdot 1 \cdot 2\pi 60 \cdot \frac{0.74857 \cdot 10^{-6}}{0.476 \cdot 10^{-3}} \cdot (-712.088 + j2030.4 + 1724.25 + j853) \\ &= -1735.75 + j2389.4A \end{aligned}$$

$$|I_6| = 2953.31A$$

Now the total current

$$\begin{aligned} I_b' &= I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = -712.088 + j2030.4 - 1735.75 + j2389 = \\ &= -2447.75 + j4419.4A \end{aligned}$$

$$I_b' \approx 5050A$$

The a.c. power in the bar is

$$\begin{aligned} P_{ac} &= \sum_{j=1}^n I_j^2 R_j = 0.333 \cdot 10^{-3} (600^2 + 636.74^2 + 827.30^2) + \\ &+ 0.833 \cdot 10^{-3} \cdot 493.27^2 + 0.476 \cdot 10^{-3} (2151.2^2 + 2955.31^2) = \\ &= 482 + 202.68 + 6360 = 7044.68W \end{aligned}$$

The d.c. current distribution in the 6 layer is uniform, therefore

$$I_{1dc} = I_{2dc} = I_{3dc} = \frac{I_b'}{A_{bar}} b_1 h_t = \frac{5050 \cdot 20.5}{(14.10 + 8.5 + 15.20)} = 1052.08A$$

$$I_{4dc} = \frac{I_b'}{A_{bar}} b_4 h_t = \frac{5050 \cdot 8.5}{480} = 420.83A$$

$$I_{5dc} = I_{6dc} = \frac{I_b'}{A_{bar}} b_5 h_t = \frac{5050 \cdot 14.5}{480} = 736.458A$$

$$P_{dc} = \sum_{j=1}^n I_{jdc}^2 R_j = 0.333 \cdot 10^{-3} \cdot 3 \cdot 1052.08^2 + \\ + 0.833 \cdot 10^{-3} \cdot 420.83^2 + 0.476 \cdot 10^{-3} \cdot 2 \cdot 736.458^2 = 1768.81W$$

and the skin effect resistance correction factor K_R is

$$K_R = \frac{P_{ac}}{P_{dc}} = \frac{7044.68}{1768.81} = 3.9827!$$

The magnetic energy ratio K_x is

$$K_x = \frac{A}{B}$$

$$A = L_1 \left(|I_1|^2 + |I_1 + I_2|^2 + |I_1 + I_2 + I_3|^2 \right) + L_4 |I_1 + I_2 + I_3|^2 + \\ + L_5 |I_1 + I_2 + I_3 + I_4 + I_5|^2 + L_5 |I_1 + I_2 + I_3 + I_4 + I_5 + I_6|^2$$

$$B = L_1 \left[I_{1dc}^2 + (I_{1dc} + I_{2dc})^2 + (I_{1dc} + I_{2dc} + I_{3dc})^2 \right] + \\ + L_4 (I_{1dc} + I_{2dc} + I_{3dc} + I_{4dc})^2 + L_5 (I_b' - I_6)^2 + L_5 I_b'^2$$

$$A = 0.314 \cdot 10^{-6} (600^2 + 1.217^2 10^6 + 1.9237^2 10^6) + 0.785 \cdot 10^{-6} 2.229^2 10^6 + \\ + 0.44857 \cdot 10^{-6} (3.0554^2 10^6 + 5.050^2 10^6)$$

$$B = 0.314 \cdot 10^{-6} (1.052^2 10^6 + 2.104^2 10^6 + 3.156^2 10^6) + \\ + 0.785 \cdot 10^{-6} 3.576^2 10^6 + 0.44857 \cdot 10^{-6} (4.3128^2 10^6 + 5.05^2 10^6)$$

$$K_x = \frac{21.267}{34.685} = 0.613!$$

The inductance coefficient refers only to the slot body (filled with conductor) and not to the slot neck, if any.

A few remarks are in order.

- The distribution of current in the various layers is nonuniform when the skin effect occurs.
- Not only the amplitude, but the phase angle of bar current in various layers varies due to skin effect ([Figure 9.14](#)).
- At $S = 1$ ($f_1 = 60$ Hz) most of the current occurs in the upper part of the slot.

- The equivalent circuit model can be easily put into computer form once the layers geometry— h_t (height) and b_j (width)—are given. For various practical slots special subroutines may provide b_j , h_t when the number of layers is given.
- To treat a double cage by this method, we have only to consider zero the current in the empty slot layers between the upper and lower cage (Figure 9.15).

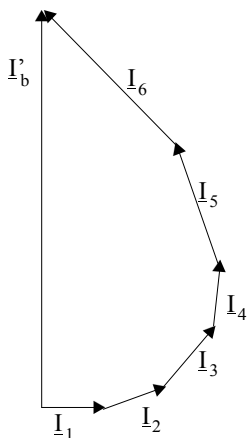


Figure 9.14 Layer currents and the bar current with skin effect

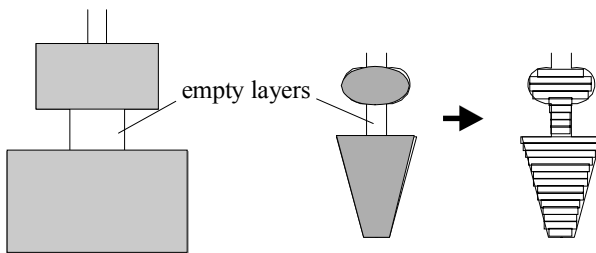


Figure 9.15 Treating skin effect with equivalent circuit (or multilayer) method

Now that both K_R and K_x are known, the bar resistance and slot body leakage geometrical specific permeance λ_{sbody} is modified to account for skin effect.

$$(\lambda_{sbody})_{ac} = (\lambda_{sbody})_{dc} K_x \quad (9.45)$$

From d.c. magnetic energy W_{mdc} (9.42), we write

$$L_{dc} = \frac{1}{2} \frac{W_{mdc}}{I_b^2} = \mu_0 l_{stack} (\lambda_{sbody})_{dc} \quad (9.46)$$

The slot neck geometrical specific permeance is still to be added to account for the respective slot leakage flux. This slot neck geometrical specific permeance is to be corrected for leakage flux saturation discussed later in this chapter.

9.4. SKIN EFFECT IN THE END RINGS VIA THE MULTILAYER APPROACH

As the end rings are placed in air, although rather close to the motor laminated stack, the skin effect in them is routinely neglected. However, there are applications where the value of slip goes above unity ($S = \text{up to } 3.0$ in standard elevator drives) or the slip frequency is large as in high frequency (high speed) motors to be started at rated frequency (400 Hz in avionics).

For such cases, the multilayer approach may be extended to end rings. To do so we introduce radial and circumferential layers in the end rings (Figure 9.16) as shown in Reference [7].

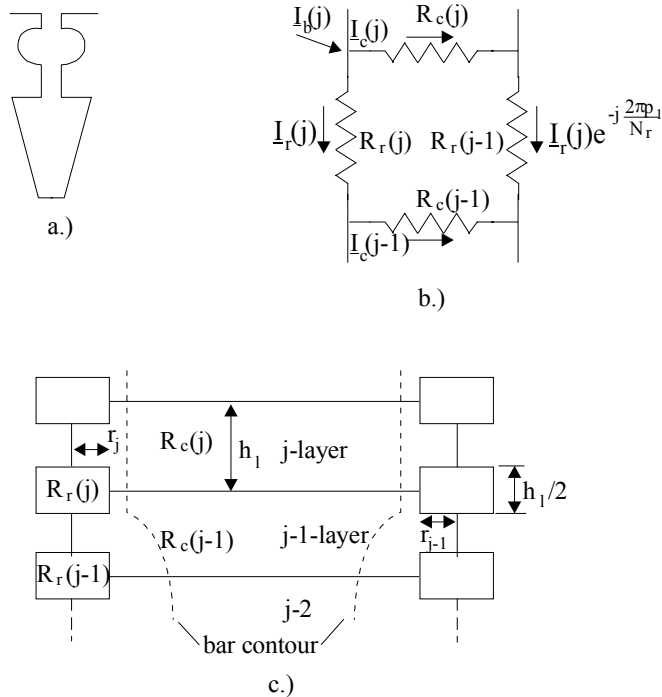


Figure 9.16 Bar-end ring transition
a.) slot cross – section b.) radial and circumferential layer end ring currents
c.) geometry of radial and circumferential end ring layers

In all layers, the current density is considered uniform. It means that their radial dimension has to be less than the depth of field penetration in aluminum.

The currents in neighboring slots are considered phase-shifted by $2\pi p_1/N_r$ radians (N_r —number of rotor slots).

The relationship between bar and end ring layer currents (Figure 9.16) is

$$I_{b(j)} + I_{r(j+1)} + I_{r(j)} e^{j\frac{2\pi p_1}{N_r}} = I_{r(j)} + I_{c(j)} \quad (9.47)$$

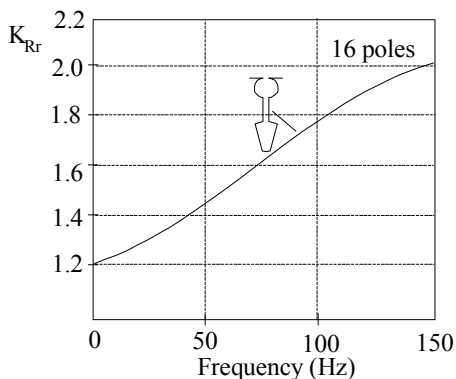


Figure 9.17 End ring skin effect resistance coefficient K_{Rr}

$$R_{c(j)} I_{c(j)} + R_{r(j)} I_{r(j)} e^{-j\frac{2\pi p_1}{N_r}} = R_{c(j-1)} I_{c(j-1)} + R_{r(j)} I_{r(j)} \quad (9.48)$$

The circumferential extension of the radial layer r_j is assigned a value at start. Now if we add the equations for the bar layer currents, we may solve the system of equations. As long as the radial currents increase, γ_j is increased in the next iteration cycle until sufficient convergence is met. Some results, after [2], are given in Figure 9.17.

As the slot total height is rather large (above 25 mm), the end ring skin effect is rather large, especially for rotor frequencies above 50(60) Hz. In fact, a notable part of this resistance rise is due to the radial ring currents which tend to distribute the bar currents, gathered toward the slot opening, into most of end ring cross section.

9.5. THE DOUBLE CAGE BEHAVES LIKE A DEEP BAR CAGE

In some applications, very high starting torque— $T_{start}/T_{rated} \geq 2.0$ —is required. In such cases, a double cage is used. It has been proved that it behaves like a deep bar cage, but it produces even higher starting torque at lower starting current. For the case when skin effect can be neglected in both cages, let us consider a double cage as configured in Figure 9.18. [8]

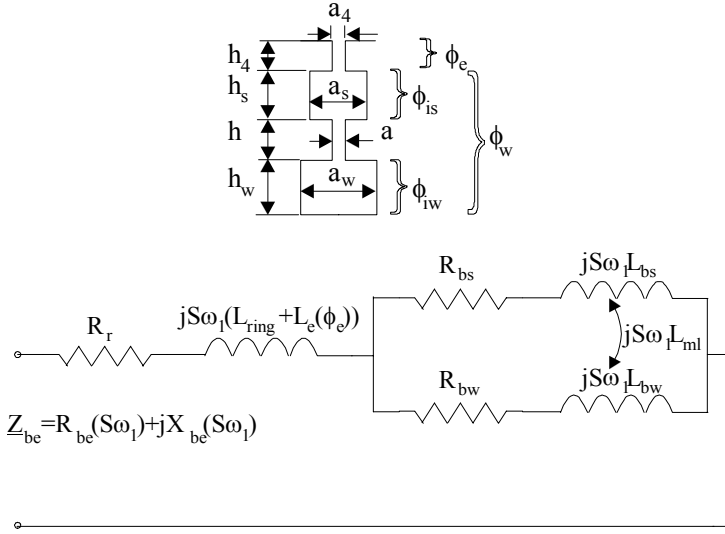


Figure 9.18 Double cage rectangular-shape geometry a.) and equivalent circuit b.)

The equivalent single bar circuit is given in [Figure 9.18b](#). For the common ring of the two cages

$$R_r = R_{ring}, R_{bs} = R_{bs \text{ upper bars}}, R_{bw} = R_{bw \text{ lower bar}} \quad (9.49)$$

For separate rings

$$R_r = 0, R_{bs} = R_{bs} + R_{rings}^e, R_{bw} = R_{bw} + R_{ringw}^e \quad (9.50)$$

The ring segments are included into the bar resistance after approximate reduction as shown in Chapter 6. The value of L_{ring} is the common ring inductance or is zero for separate rings. Also for both cases, $L_e(\Phi_e)$ refers to the slot neck flux.

$$L_e(\Phi_e) = \mu_0 I_{stack} \frac{h_4}{a_4} \quad (9.51)$$

We may add into L_e the differential leakage inductance of the rotor.

The start (upper) and work (lower) cage inductances L_{bs} and L_{bw} include the end ring inductances only for separate rings. Otherwise, the bar inductances are

$$L_{bs} = \mu_0 I_{stack} \frac{h_s}{3a_s} \quad (9.52)$$

$$L_{bw} = \mu_0 I_{stack} \left(\frac{h_w}{3a_w} + \frac{h}{a} + \frac{h_s}{a_s} \right)$$

There is also a flux common to the two cages represented by the flux in the starting cage. [3]

$$L_{ml} = \mu_0 l_{stack} \frac{h_s}{2a_s} \quad (9.53)$$

In general, L_{ml} is neglected though it is not a problem to consider in solving the equivalent circuit in Figure 9.18. It is evident (Figure 9.18a) that the starting (upper) cage has a large resistance (R_{bs}) and a small slot leakage inductance L_{bs} , while for the working cage the opposite is true.

Consequently, at high slip frequency, the rotor current resides mainly in the upper (starting) cage while, at low slip frequency, the current flows mainly into the working (lower) cage. Thus both R_{be} and X_{be} vary with slip frequency as they do in a deep bar single cage (Figure 9.19).

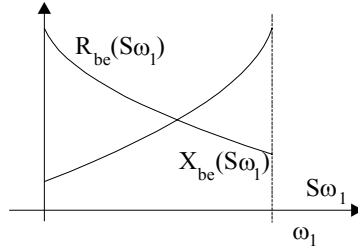


Figure 9.19 Equivalent parameters of double cage versus slip frequency

9.6. LEAKAGE FLUX PATH SATURATION—A SIMPLIFIED APPROACH

Leakage flux path saturation occurs mainly in the slot necks zone for semiclosed slots for currents above 2 to 3 times rated current or in the rotor slot iron bridges for closed slots even well below the rated current (Figure 9.20).

Consequently,

$$\begin{aligned} a_{os}' &= a_{os} + \frac{(\tau_{ss} - a_{os})}{\mu_s} \mu_0; \quad \tau_{ss} = \frac{\pi D_i}{N_s} \\ a_{or}' &= a_{or} + \frac{(\tau_{sr} - a_{or})}{\mu_r} \mu_0; \quad \tau_{sr} = \frac{\pi(D_i - 2g)}{N_s} \\ a_{or}' &= \frac{b_{or} \mu_b}{\mu_{br}} \end{aligned} \quad (9.54)$$

The slot neck geometrical permeances will be changed to: a'_{os}/h_{os} , a'_{or}/h_{or} , or a'_{or}/h_{or} dependent on stator (rotor) current.

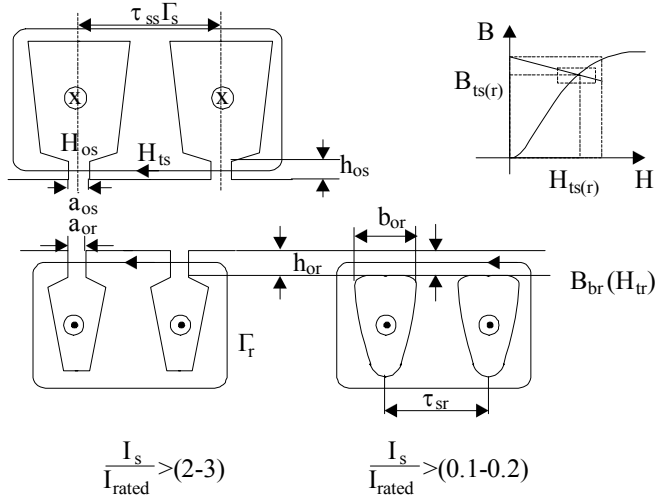


Figure 9.20 Leakage flux path saturation conditions

With n_s the number of turns (conductors) per slot, and I_s and I_b the stator and rotor currents, the Ampere's law on Γ_s , Γ_r trajectories in Figure 9.20 yields

$$\begin{aligned} H_{ts}(\tau_{ss} - a_{os}) + H_{os}a_{os} &\approx n_s I_s \sqrt{2}; \quad \mu_s H_{ts} = B_{ts} = \mu_0 H_{os} \\ H_{tr}(\tau_{sr} - a_{or}) + H_{or}a_{or} &\approx I_b \sqrt{2}; \quad \mu_r H_{tr} = B_{tr} = \mu_0 H_{or} \\ H_{tr}b_{or} &= I_b \sqrt{2}; \quad \mu_{br} = \frac{B_{tb}}{H_{tb}} \end{aligned} \quad (9.55)$$

The relationship between the equivalent rotor current I_r' (reduced to the stator) and I_b is (Chapter 8)

$$I_b = I_r' \frac{6p_1 q n_s K_{wl}}{N_r} = I_r' n_s K_{wl} \frac{N_s}{N_r} \quad (9.56)$$

N_s —number of stator slots; K_{wl} —stator winding factor.

When the stator and rotor currents I_s and I_r' are assigned pertinent values, iteratively, using the lamination magnetization curves, Equations (9.55) may be solved (Figure 9.20) to find the iron permeabilities of teeth tops or of closed rotor slot bridges. Finally, from (9.54), the corrected slot openings are found.

With these values, the stator and rotor parameters (resistances and leakage inductances) as influenced by the skin effect (in the slot body zone) and by the leakage saturation (in the slot neck permeance) are recalculated. Continuing with these values, from the equivalent circuit, new values of stator and rotor currents I_s , I_r' are calculated for given stator and voltage, frequency and slip.

The iteration cycles continue until sufficient convergence is obtained for stator current.

Example 9.4. An induction motor has semiclosed rectangular slots whose geometry is shown in [Figure 9.21](#).

The current density design for rated current is $j_{Co} = 6.5 \text{ A/mm}^2$ and the slot fill factor $K_{fill} = 0.45$. The starting current is 6 times rated current. Let us calculate the slot leakage specific permeance at rated current and at start.

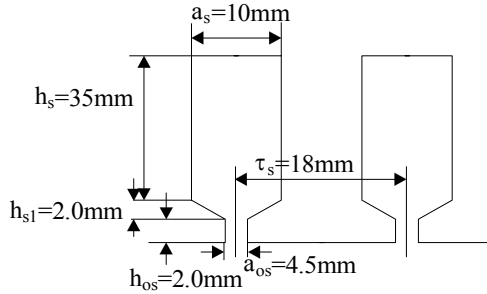


Figure 9.21 Slot geometry

Solution

The slot mmf $n_s I_s$ is obtained from

$$n_s I_s = (h_s a_s) K_{fill} j_{Co} = 10 \cdot 35 \cdot 0.45 \cdot 6.5 = 1023.75 \text{ Ampereturns}$$

Making use of (9.54),

$$\left[H_{ts} (18 - 4.5) + \frac{B_{ts}}{\mu_0} 4.5 \right] \cdot 10^{-3} = 1023.75 \sqrt{2}$$

$$B_{ts} = \left(1023.75 \sqrt{2} - H_{ts} \cdot 13.5 \cdot 10^{-3} \right) \frac{1.256 \cdot 10^{-6}}{4.5 \cdot 10^{-3}} = 0.4029 - H_{ts} 3\mu_0$$

It is very clear that for rated current $B_{ts} < 0.4029$, so the tooth heads are not saturated. In (9.54), $a_{os}' = a_{os}$ and the slot specific geometrical permeance λ_s is

$$\lambda_s = \frac{h_s}{3a_s} + \frac{2h_{sl}}{a_{os} + a_s} + \frac{h_{os}}{a_{os}} = \frac{35}{3 \cdot 10} + \frac{2 \cdot 2.5}{4.5 + 10} + \frac{2}{4.5} = 1.9559$$

For the starting conditions,

$$\left[H_{tstart} (18 - 4.5) \cdot 10^{-3} + \frac{B_{ts}}{\mu_0} 4.5 \right] \cdot 10^{-3} = 6142.5 \sqrt{2}$$

$$B_{tstart} = 2.41 - H_{tstart} 3\mu_0$$

Now we need the lamination magnetization curve. If we do so we get $B_{tstart} \approx 2.16 \text{ T}$, $H_{tstart} \approx 70,000 \text{ A/m}$. The iron permeability μ_s is

$$\frac{\mu_s}{\mu_0} = \frac{B_{\text{tstart}}}{H_{\text{tstart}}} \mu_0^{-1} = \frac{2.16}{70000 \cdot 1.256 \cdot 10^{-6}} = 24.56$$

$$\text{Consequently, } a_{os}' = a_{os} + \frac{(\tau_s - a_{os})}{\mu_s} \mu_0 = 4.5 + \frac{(18 - 4.5)}{24.56} = 5.05 \text{ mm}$$

The slot geometric specific permeance is, at start,

$$\lambda_s' = \frac{h_s}{3a_s} + \frac{2h_{s1}}{a_{os}' + a_s} + \frac{h_{os}}{a_{os}'} = \frac{35}{3 \cdot 10} + \frac{2 \cdot 2.5}{5.05 + 10} + \frac{2}{5.05} = 1.895$$

The leakage saturation of tooth heads at start is not very important in our case. In reality, it could be more important for semiclosed stator (rotor) slots.

Example 9.5. Let us consider a rotor bar with the geometry of Figure 9.21. The bar current at rated current density $j_{Alr} = 6.0 \text{ A/mm}^2$ is

$$I_{br} = j_{Alr} h_r \frac{(a_r + b_r)}{2} = 25 \frac{(11 + 5)}{2} \cdot 6 = 1200 \text{ A}$$

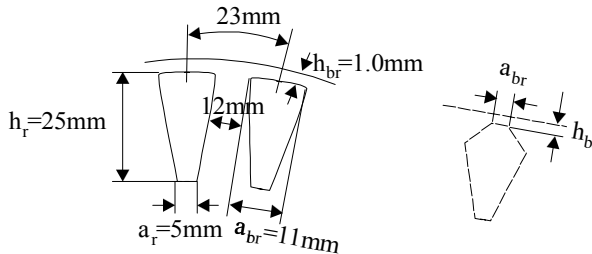


Figure 9.22. Closed rotor slots

Calculate the rotor bridge flux density for 10%, 20%, 30%, 50%, 100% of rated current and the corresponding slot geometrical specific permeance.

Solution

Let us first introduce a typical magnetization curve.

B(T)	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
H(A/m)	22.8	35	45	49	57	65	70	76	83	90
B(T)	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
H(A/m)	98	106	115	124	135	148	162	177	198	220

B(T)	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
H(A/m)	237	273	310	356	417	482	585	760	1050	1340
B(T)	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9	1.95	2.0
H(A/m)	1760	2460	3460	4800	6160	8270	11170	15220	22000	34000

Neglecting the tooth top saturation and including only the bridge zone, the Ampere's law yields

$$H_{br} a_{br} = I_b; \quad H_{br} = \frac{I_b}{a_{br}}; \quad a_{br} = 11\text{mm}$$

For various levels of bar current, H_{br} and B_{br} become

I_b (A)	120	240	360	600	1200
H_{br}	10909	21818	32727	54545	109090
B_{br}	1.84	1.95	1.98	2.1	2.2
$\mu_{rel} = \frac{\mu_{br}}{\mu_0}$	134.28	71.16	48.165	30.653	16.056
$a_{or}' = \frac{a_{br}}{\mu_{rel}}$ (mm)	0.0819	0.1546	0.2284	0.3588	0.6851
$\lambda_{slot} = \frac{2h_r}{3(a_r + a_{br})} + \frac{h_{br}}{a_{or}'}$	14.293	75.1	5.42	3.8287	2.5013

Note that at low current levels the slot geometrical specific permeance is unusually high. This is because the iron bridge is long ($a_{br} = 11\text{mm}$); so H_{br} is small at small currents.

Also, the iron bridge is 1mm thick in our case, while 0.5 to 0.6 mm would produce better results (lower λ_{slot}). The slot height is quite large ($h_r = 25\text{ mm}$) so significant skin effect is expected at high slips. We investigated here only the currents below the rated one so $Sf_1 < 3\text{ Hz}$ in general. Should the skin effect occur, it would have reduced the first term in λ_{slot} (slot body specific geometrical permeance) by the factor K_x .

As a conclusion to this numerical example, we may infer that a thinner bridge with a smaller length would saturate sooner (at smaller currents below the rated one), producing finally a lower slot leakage permeance λ_{slot} . The advantages of closed slots are related to lower stray load losses and lower noise.

9.7. LEAKAGE SATURATION AND SKIN EFFECTS—A COMPREHENSIVE ANALYTICAL APPROACH

Magnetic saturation in induction machines occurs in the main path, at low slip frequencies and moderate currents unless closed rotor slots are used when their iron bridges saturate the leakage path.

In contrast, for high slip frequencies and high currents, the leakage flux paths saturate as the main flux decreases with slip frequency for constant stator voltage and frequency.

The presence of slot openings, slot skewing, and winding distribution in slots, make the problem of accounting for magnetic saturation, in presence of rotor skin effects, when the induction machine is voltage fed, a very difficult task.

Ultimately a 3D finite element approach would solve the problem, but for prohibitive computation time.

Less computation, intensive, analytical solutions have been gradually introduced [9–15]. However, only the starting conditions have been treated (with $I_s = I_r'$ known), with skin effect neglected. Main flux path saturation level, skewing, zig-zag leakage flux have all to be considered for a more precise assessment of induction motor parameters for large currents.

In what follows, an attempt is made to allow for magnetic saturation in the main and leakage path (with skin effect accounted for by appropriate correction coefficients K_R , K_X) for given slip, frequency, voltage, motor geometry, no-load curve, by iteratively recalculating the stator magnetization and rotor currents until sufficient convergence is obtained. So, in fact, the saturation and skin effects are explored from no-load to motor stall conditions.

The computation algorithm presupposes that the motor geometry and the no-load curve as $L_m(I_m)$, magnetising inductance versus no-load current, is known based on analytical or FEM modeling or from experiments.

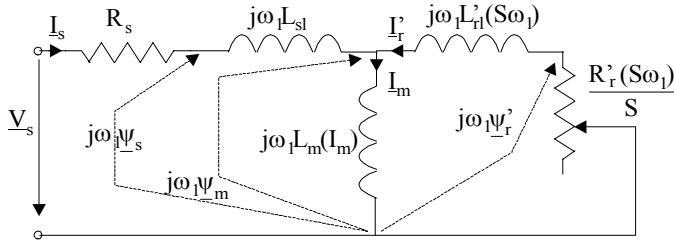


Figure 9.23 Equivalent circuit with main flux path saturation and skin effects accounted for

The leakage inductances are initially considered unsaturated (as derived in Chapter 6) with their standard expressions. The influence of skin effect on slot-body specific geometric permeance and on resistances is already accounted for in these parameters.

Consequently, given the slip value, the equivalent circuit with the variable parameter $L_m(I_m)$, Figure 9.23, may be solved.

The phasor diagram corresponding to the equivalent circuit in Figure 9.23 is shown in Figure 9.24.

From the equivalent circuit, after a few iterations, based on the $L_m(I_m)$ saturation function, we can calculate I_s , ϕ_s , I_r' , ϕ_r , I_m , ϕ_m .

As expected, at higher slip frequency ($S\omega_l$) and high currents, the main flux ψ_m decreases for constant voltage V_s and stator frequency ω_l . At start ($S = 1$, rated voltage and frequency), due to the large voltage drop over $(R_s + j\omega_l L_{sl})$, the main flux ψ_m decreases to 50 to 60% of its value at no-load.

This ψ_m decreasing with slip and current rise led to the idea of neglecting the main flux saturation level when leakage flux path saturation was approached. However, at a more intrusive view, in the teeth top main flux (Ψ_m), slot neck leakage flux (Φ_{st1} , Φ_{st2}), zig-zag leakage flux (Φ_{z1} , Φ_{z2}), the so-called

[illegible]

where λ_m is the equivalent main flux geometrical specific permeance.

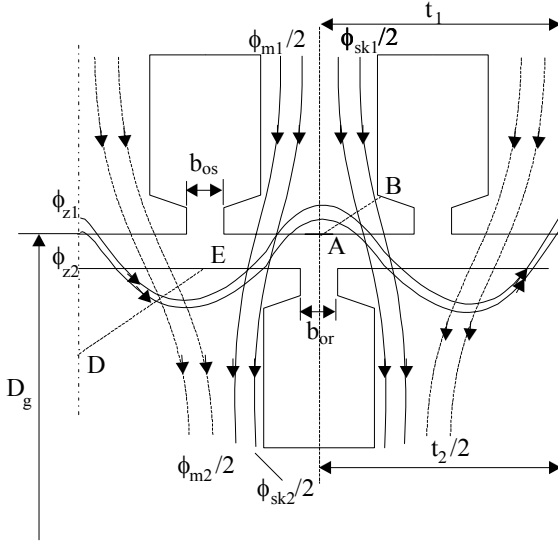


Figure 9.25 Computational flux lines

In a similar manner, the zig-zag fluxes Φ_{z1} and Φ_{z2} are

$$\begin{aligned}\Phi_{z1} &= AT_1 \mu_0 l_{\text{stack}} \lambda_{z1} \\ \Phi_{z2} &= AT_2 \mu_0 l_{\text{stack}} \lambda_{z2} \\ AT_1 &= n_s I_s \sqrt{2}; \quad AT_2 = n_r I'_r \sqrt{2};\end{aligned}\tag{9.59}$$

From (9.56) and (9.59) n_r is

$$n_r = \frac{I_b \sqrt{2}}{I'_r \sqrt{2}} = n_s K_{w1} \frac{N_s}{N_r}\tag{9.60}$$

The unsaturated zig-zag specific permeances λ_{z1} and λ_{z2} are [5]

$$\lambda_{z1} = \frac{K_{w1}^2 \pi D_g}{12 N_s g} (1.2 a_1 - 0.2)\tag{9.61}$$

$$\lambda_{z2} = \frac{K_{w1}^2 \pi D_g}{12 N_r g} (1.2 a_2 - 0.2)\tag{9.62}$$

$$a_1 = \frac{t_1 (5g + b_{os}) - b_{or}^2}{t_1 (5g + b_{os})}\tag{9.63}$$

$$a_2 = \frac{t_2(5g + b_{or}) - b_{os}^2}{t_2(5g + b_{or})} \quad (9.64)$$

The slot neck fluxes Φ_{st1} and Φ_{st2} are straightforward.

$$\Phi_{st1} = AT_1\mu_0 l_{stack}\lambda_{st1}; \quad \lambda_{st1} = \frac{h_{os}}{b_{os}} \quad (9.65)$$

$$\Phi_{st2} = AT_2\mu_0 l_{stack}\lambda_{st2}; \quad \lambda_{st2} = \frac{h_{or}}{b_{or}} \quad (9.66)$$

The differential leakage, specific permeance $\lambda_{d1,2}$ may be derived from a comparison between magnetization harmonic inductances L_{mv} and the leakage inductance expressions L_{sl} and L_{rl}' (Chapter 7).

$$L_{mv} = \frac{6\mu_0}{\pi^2} \frac{(\tau/v)l_{stack}(W_1K_{wv})^2}{(p_1v)gK_c(1+K_{steeth})} \quad (9.67)$$

$$L_{sl} = 2\mu_0 \frac{W_1^2}{p_1q} (\lambda_{ss} + \lambda_{z1} + \lambda_{endcon1} + \lambda_{st1} + \lambda_{d1}) \quad (9.68)$$

$$L_{sr} = 2\mu_0 \frac{W_1^2}{p_1q} (\lambda_{sr} + \lambda_{z2} + \lambda_{endcon2} + \lambda_{d2} + \lambda_{st2} + \lambda_{sk2}) \quad (9.69)$$

$$\begin{aligned} \lambda_{d1} &= \sum_{v>1}^{\infty} 3 \frac{K_{wv}^2}{gK_c v^2} \frac{q_1 \tau}{(1+K_{steeth})} \approx \lambda_{m1} (\tau_{d1} - \Delta\tau_{d1}) \\ \tau_{d1} &= \pi^2 \frac{(10q_1^2 + 2)}{27} \sin^2 \left(\frac{30^0}{q_1} \right) - 1; \quad \Delta\tau_{d1} = K_{z1} \frac{1}{9q_1^2} \\ \lambda_{d2} &= \lambda_{m1} \frac{N_s}{N_r} (\tau_{d2} - \Delta\tau_{d2}); \quad \Delta\tau_{d2} = K_{z2} \left(\frac{2p_1}{N_r} \right)^2 \\ \tau_{d2} &= \frac{\left(\frac{\pi p_1}{N_r} \right)^2}{\sin^2 \left(\frac{\pi p_1}{N_r} \right)} \frac{\left(\frac{\pi p_1 \cdot skew}{N_r} \right)^2}{\sin^2 \left(\frac{\pi p_1 \cdot skew}{N_r} \right)} - 1 \end{aligned} \quad (9.70)$$

The coefficient K_{z1} and K_{z2} are found from [Figure 9.26](#).

The slot-specific geometric permeances λ_{ss} and λ_{sr} , for rectangular slots, with rotor skin effect, are

$$\lambda_{ss} = \frac{h_s}{3b_s} + \lambda_{st1} \quad (9.71)$$

$$\lambda_{sr} = \frac{h_r}{3b_r} \cdot K_x + \lambda_{st2}; \quad K_x - \text{skin effect correction} \quad (9.72)$$

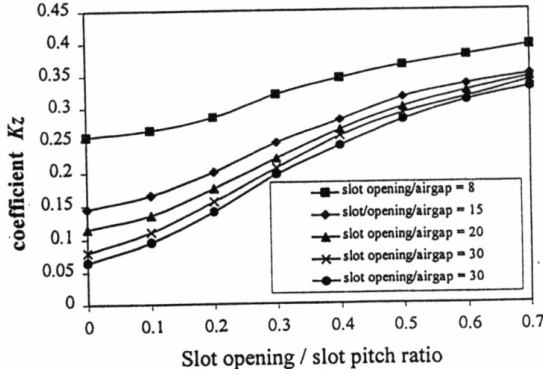


Figure 9.26 Correction coefficient for differential leakage damping

The most important “ingredient” in leakage inductance saturation is, for large values of stator and rotor currents, the skewing (uncompensated) mmf.

9.7.1. The skewing mmf

The rotor (stator) slot skewing, performed to reduce the first slot harmonic torque (in general) has also some side effects such as: a slight reduction in the magnetization inductance L_m accompanied by an additional leakage inductance component, L'_{rskew} . We decide to “attach” this new leakage inductance to the rotor as it “disappears” at zero rotor current.

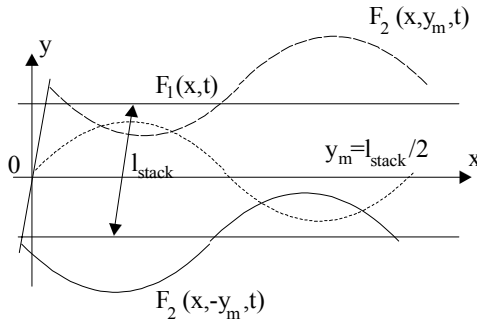


Figure 9.27. Phasing of stator and rotor mmfs for skewed rotor slots

To account for L_m alteration due to skewing, we use the standard skewing factor $K_{\text{skew}}^{\text{conv}}$.

$$K_{\text{skew}}^{\text{conv}} = \frac{\sin\left(\frac{\pi}{2} \text{skew} / m_1 q_1\right)}{\frac{\pi}{2} \text{skew} / m_1 q_1}; \quad K_{\text{skew}}^{\text{conv}} \approx 0.98 - 1.0 \quad (9.73)$$

and include a rotor current linear dependence to make sure that the influence disappears at zero rotor current,

$$K_{\text{skew}} = 1 - \left(1 - K_{\text{skew}}^{\text{conv}}\right) \frac{I_r'}{I_s} \quad (9.74)$$

skew—is the skewing in stator slot pitch counts.

To account for the skewing effect, we first consider the stator and rotor mmf fundamentals for a skewed rotor.

As the current in the bar is the same all along the stack length (with interbar currents neglected), the rotor mmf phasing varies along OY, beside OX and time (Figure 9.27) so the resulting mmf fundamental is

$$\begin{aligned} F(x, y, t) = & F_{1m} \sin\left(\frac{\pi}{\tau} x - 2\pi f_1 t\right) + \\ & + F_{2m} \sin\left(\frac{\pi}{\tau} x - 2\pi f_1 t - (\varphi_s - \varphi_r) - \frac{\pi}{m_1 q_1} \text{skew} \frac{y}{l_{\text{stack}}}\right) \\ & - \frac{l_{\text{stack}}}{2} \leq y \leq \frac{l_{\text{stack}}}{2} \end{aligned} \quad (9.75)$$

We may consider this as made of two terms, the magnetization mmf $F_{1\text{mag}}$ and the skewing mmf $F_{2\text{skew}}$.

$$F(x, y, t) = F_{1\text{mag}} + F_{2\text{skew}} \quad (9.76)$$

$$\begin{aligned} F_{1\text{mag}} = & F_{1m} \sin\left(\frac{\pi}{\tau} x - 2\pi f_1 t\right) + \\ & + F_{2m} \cos\left(\frac{\pi}{m_1 q_1} \text{skew} \frac{y}{l_{\text{stack}}}\right) \cdot \sin\left(\frac{\pi}{\tau} x - 2\pi f_1 t - (\varphi_s - \varphi_r)\right) \end{aligned} \quad (9.77)$$

$$F_{2\text{skew}} = -F_{2m} \sin\left(\frac{\pi}{m_1 q_1} \text{skew} \frac{y}{l_{\text{stack}}}\right) \cdot \cos\left(\frac{\pi}{\tau} x - 2\pi f_1 t - (\varphi_s - \varphi_r)\right) \quad (9.78)$$

$F_{1\text{mag}}$ is only slightly influenced by skewing (9.77) as also reflected in (9.73).

On the other hand, the skewing (uncompensated) mmf varies with rotor current (F_{2m}) and with the axial position y . It tends to be small even at rated

current in amplitude (9.78) but it gets rather high at large rotor currents. Notice also that the phase angle of $F_{1\text{mag}}$ (φ_m) and $F_{2\text{skew}}$ (φ_r) are different by slightly more than 90° (Figure 9.24), above rated current,

$$\varphi_{\text{skew}} = \varphi_r - \varphi_m \quad (9.79)$$

We may now define a skewing magnetization current I_{mskew} :

$$I_{\text{mskew}} = \pm I_m \frac{AT_2}{AT_{1m}} \sin(\pi 2p_1 SKW_1 / N_s) \quad (9.80)$$

Example 9.6. The skewing mmf

Let us consider an induction motor with the following data: no-load rated current: $I_m = 30\%$ of rated current; starting current. 600% of rated current; the number of poles: $2p_1 = 4$, skew = 1 stator slot pitch; number of stator slots $N_s = 36$. Assessment of the maximum skewing magnetization current I_{mskew}/I_m for starting conditions is required.

Solution

Making use of (9.80), the skewing magnetization current for $SKW_1 = \pm \text{skew}$.

$$\begin{aligned} \frac{I_{\text{mskew}}}{I_m} &= \pm \frac{AT_2}{AT_{1m}} \sin\left(\frac{\pi 2p_1 (\text{skew} / 2)}{N_s}\right) = \pm \left(\frac{I_r'}{I_m}\right) \sin\left(\frac{\pi \cdot 4 \cdot 1 / 2}{36}\right) \approx \\ &\approx \pm \left(\frac{I_{\text{start}}}{I_m}\right) \cdot 0.15643 = \pm \frac{600\%}{30\%} \cdot 0.15643 = \pm 3.1286! \end{aligned}$$

This shows that, at start, the maximum flux density in the airgap produced by the skewing magnetization current in the extreme axial segments will be maximum and much higher than the main flux density in the airgap, which is dominant below rated current when I_{mskew} becomes negligible (Figure 9.28).

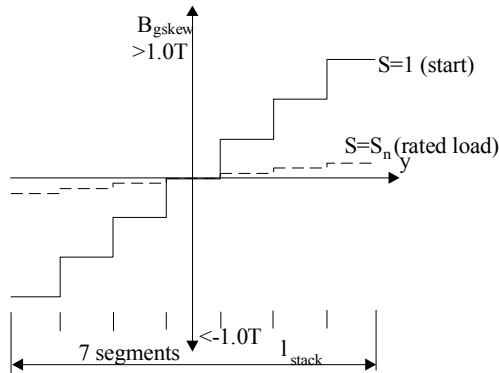


Figure 9.28 Skewing magnetization current produced airgap flux density

Airgap flux densities in excess of 1.0 T are to be expected at start and thus heavy saturation all over magnetic circuit should occur.

Let us remember that in these conditions the magnetization current is reduced such that the airgap flux density of main flux is (0.6 – 0.65) of its rated value, and its maximum is phase shifted with respect to the skewing flux by more than 90° (Figure 9.24). Consequently at start (high currents), the total airgap flux density is dominated by the skewing mmf contribution! This phenomenon has been documented recently through FEM [16].

Dividing the stator stack in ($N_{\text{seg}} + 1$) axial segments, we have:

$$SKW_1 = 0.0, \frac{\text{skew}}{N_{\text{seg}}}, \frac{\text{skew}}{2} \quad (9.81)$$

Now, based on I_{mskew} , from the magnetization curve ($L_m(I_m)$), we may calculate the magnetization inductance for skewing magnetization current $L_m(I_{\text{mskew}})$. Finally we may define a specific geometric permeance for skewing λ_{sk1} .

$$\lambda_{\text{sk1}} = \frac{L_m(I_{\text{mskew}})}{N_s K_{w1} W_1 n_s \mu_0 l_{\text{stack}}} \quad (9.82)$$

W_1 —turns/phase.

λ_{sk1} depends on the segment considered (6 to 10 segments are enough) and on the level of rotor current.

Now the skewing flux/tooth in the stator and rotor is

$$\Phi_{\text{sk1}} = AT_2 \sin\left(\frac{\pi 2 p_1 SKW_1}{N_s}\right) \mu_0 l_{\text{stack}} \lambda_{\text{sk1}} \quad (9.83)$$

$$\Phi_{\text{sk2}} = \Phi_{\text{sk1}} \frac{N_s}{N_r} \quad (9.84)$$

In (9.83), the value of Φ_{sk1} is calculated as if the value of segmented skewing is valid all along the stack length. This means that all calculations will be made n_{seg} times and then average values will be used to calculate the final values of leakage inductances.

Now if the skewing flux occurs both in the stator and in the rotor, when the leakage inductance is calculated, the rotor one will include the skewing permeance λ_{sk2} .

$$\lambda_{\text{sk2}} = \lambda_{\text{sk1}} \frac{N_s}{N_r} \quad (9.85)$$

9.7.2. Flux in the cross section marked by AB (Figure 9.25)

The total flux through AB, Φ_{AB} is

Equation (9.92) corroborated with the lamination $B_l(H_l)$ magnetization curve leads to B_l and H_l (Figure 9.30).

An iterative procedure as in Figure 9.30 may be used to solve (9.92) for B_l and H_l . Finally, from (9.90), the saturated value of $(\Phi_{AB})_{sat}$ is obtained.

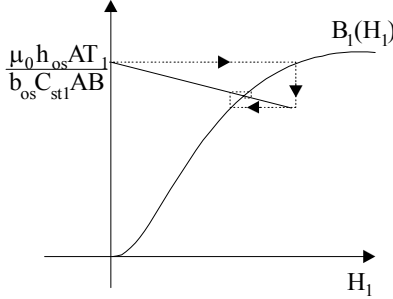


Figure 9.30 Tooth top flux density

Further on, new values of various teeth flux components are obtained.

$$\begin{aligned} \left(\frac{\Phi_{ml}}{2} \right)_{sat} &= C_{ml} (\Phi_{AB})_{sat} \\ (\Phi_{skl})_{sat} &= C_{skl} (\Phi_{AB})_{sat} \\ (\Phi_{zl})_{sat} &= C_{zl} (\Phi_{AB})_{sat} \\ (\Phi_{stl})_{sat} &= C_{stl} (\Phi_{AB})_{sat} \end{aligned} \quad (9.93)$$

The new (saturated) values of stator leakage permeance components are

$$\begin{aligned} (\lambda_{stl})_{sat} &= \frac{(\Phi_{stl})_{sat}}{AT_l \mu_0 l_{stack}} \\ (\lambda_{zl})_{sat} &= \frac{(\Phi_{zl})_{sat}}{AT_l \mu_0 l_{stack}} \end{aligned} \quad (9.94)$$

The end connection and differential geometrical permeances λ_{endcon} and λ_{dl} maintain, in (9.68), their unsaturated values, while $(\lambda_{stl})_{sat}$ and $(\lambda_{zl})_{sat}$ (via (9.94)) enter (9.68) and thus the saturated value of stator leakage inductance L_{sl} for given stator and rotor currents and slip is obtained.

We have to continue with the rotor. There may be two different situations.

9.7.4. Unsaturated rotor tooth top

In this case the saturation flux from the stator tooth is “translated” into the rotor by the ratio of slot numbers N_s/N_r .

$$\begin{aligned} (\lambda_{st2})_{sat} &= \frac{(\Phi_{st1})_{sat} \frac{N_s}{N_r}}{AT_l \mu_0 l_{stack}} \\ (\lambda_{z2})_{sat} &= \frac{(\Phi_{z1})_{sat} \frac{N_s}{N_r}}{AT_l \mu_0 l_{stack}} \\ (\lambda_{sk2})_{sat} &= \lambda_{sk1} \frac{N_s}{N_r} \end{aligned} \quad (9.95)$$

In Equation (9.95) it is recognised that the skewing flux “saturation” is not much influenced by the tooth top flux zone saturation as it represents a small length flux path in comparison with the main flux path available for skewing flux.

By now, with (9.69), the saturated rotor leakage inductance L_{rl}' may be calculated. With these new, saturated, leakage inductances, new stator, magnetization, and rotor currents are calculated.

The process is repeated until sufficient convergence is obtained with respect to the two leakage inductances. A new slip is then chosen and the computation cycle is repeated.

9.7.5. Saturated rotor tooth tip

In this case, after the AB zone was explored as above the DE cross section is investigated (Figure 9.31). Again the Ampere’s law across tooth tip and slot neck yields

$$(t_2 - b_{or})H_2 + b_{or}H_{20} = AT_2 \quad (9.96)$$

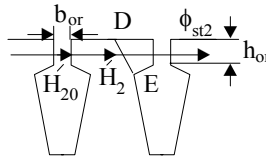


Figure 9.31 Rotor slot neck flux Φ_{st2}

The flux law results in

$$\Phi_{DE} = \Phi_1 + (\Phi_{st2})_{sat}; B_2 = \mu_0 H_{20} \quad (9.97)$$

where
$$\Phi_1 = \Phi_{ABsat} \frac{N_s}{N_r} \quad (9.98)$$

Also
$$(\Phi_{st2})_{sat} = DE \cdot l_{stack} \cdot B_2(H_2); \quad t_2' = t_2 - b_{or} \quad (9.99)$$

$$(\Phi_{st2})_{sat} = \mu_0 H_{20} h_{os} l_{stack} = \mu_0 h_{or} \frac{(AT_2 - t_2' H_2)}{b_{or}} l_{stack} \quad (9.100)$$

Finally,
$$B_2(H_2) = \frac{1}{l_{stack} DE} \left[\Phi_{ABsat} \frac{N_s}{N_r} + \mu_0 h_{or} \frac{(AT_2 - t_2' H_2)}{b_{or}} l_{stack} \right] \quad (9.101)$$

Equation (9.101), together with the rotor lamination magnetization curve, $B_2(H_2)$, is solved iteratively (as illustrated in [Figure 9.30](#) for the stator).

Then,

$$(\lambda_{st2})_{sat} = \frac{(\Phi_{st2})_{sat} \lambda_{st2}}{\Phi_{st2}}; \quad \lambda_{st2} = \frac{h_{or}}{b_{or}} \quad (9.102)$$

$$\Phi_{st2} = \mu_0 AT_2 l_{stack} \lambda_{st2} \quad (9.103)$$

The other specific geometrical permeances in the rotor leakage inductance $(\lambda_{z2})_{sat}$ and $(\lambda_{sk2})_{sat}$ are calculated as in (9.95). Now we calculate the saturated value of L_{rl}' in (9.69). Then, from the equivalent circuit, new values of currents are calculated and a new iteration is initiated until sufficient convergence is reached.

9.7.6. The case of closed rotor slots

Closed rotor slots ([Figure 9.32](#)) are used to reduce starting current, noise, vibration, and windage friction losses though the breakdown and starting torque are smaller and so is the rated power factor.

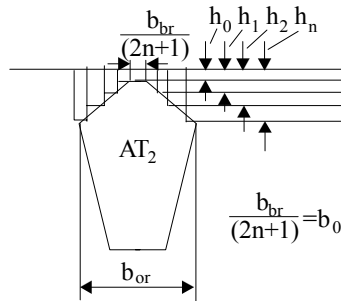


Figure 9.32 Closed rotor slot bridge discretisation

The slot iron bridge region is discretized in $2n + 1$ zones of constant (but distinct) permeability μ_j . A value of $n = (4 - 6)$ suffices. The Ampere's and flux laws for this region provide the following equations:

$$b_0 \left(H_0 + 2 \sum_{j=1}^n H_j \right) = AT_2 \quad (9.104)$$

$$B_0 h_0 = B_1 h_1 = B_2 h_2 = \dots = B_n h_n \quad (9.105)$$

Given an initial value for B_0 , for given AT_2 , $B_1, \dots, B_n$ are determined from (9.105). From the lamination magnetization curve, $H_0, H_1, \dots, H_n$ are found. A new value of AT_2 is calculated, then compared with the given one and a new B_0 is chosen. The iteration cycle is revisited until sufficient convergence is obtained.

Finally, the slot bridge geometrical specific permeance $(\lambda_{st2})_{sat}$ is

$$(\lambda_{st2})_{sat} = \frac{1}{\frac{b_0}{h_0} \frac{B_0}{\mu_0 H_0} + \frac{2b_0}{h_1} \frac{B_1}{\mu_0 H_1} + \dots + \frac{2b_0}{h_n} \frac{B_n}{\mu_0 H_n}} \quad (9.106)$$

All the other permeances are calculated as in previous paragraphs.

9.7.7. The algorithm

The algorithm for the computation of permeance in the presence of magnetic saturation and skin effects, as unfolded gradually so far, can be synthesized as following:

- Load initial data (geometry, winding data, name-plate data etc.)
- Load magnetization file: $B(H)$
- Load magnetization inductance function $L_m(I_m)$ – from FEM, design or tests
- Compute differential leakage specific permeances
- for $S = S_{min}, S_{step}, S_{max}$
 - for $SKW_1 = 0:skew/n_{seg}:skew$
 - compute unsaturated specific permeances
 - while magnetization current error $< 2\%$
 - compute induction motor model using nonlinear $L_m(I_m)$
- function
 - end;
 - Save unsaturated parameters
 - 1. if case = closed rotor slots;
 - compute closed rotor case;
 - else
 - while H_1 error $< 2\%$;
 - compute stator saturation case;
 - end;
 - compute unsaturated rotor case;

```

    if case = saturated rotor;
        while H2 error < 2%;
            compute saturated rotor case
        end;
    end.
    Compute saturated parameters; compare them with those of
    previous cycle and go to 1. Return to 1 until sufficient
    convergence in Lsl and Lrl' is reached.
end.
end (axial segments cycle)
end (slip cycle)
Save saturated parameters
for SKW1 = 0:skew/nseg:skew;
    Compute saturated parameters average values; plot the IM
    parameters (segment number, slip);
    Compute IM equivalent circuit using an average value for each
    parameter (for the nseg, axial segments);
    Compute stator, rotor, magnetization current, torque, power factor
    versus slip;
    Plot the results
end.

```

Applying this algorithm for $S = 1$ an 1.1 kW, 2 pole, three-phase IM with skewed rotor, the results shown on [Figure 9.33](#) have been obtained [18]. Heavy saturation occurs at high voltage level. The model seems to produce satisfactory results.

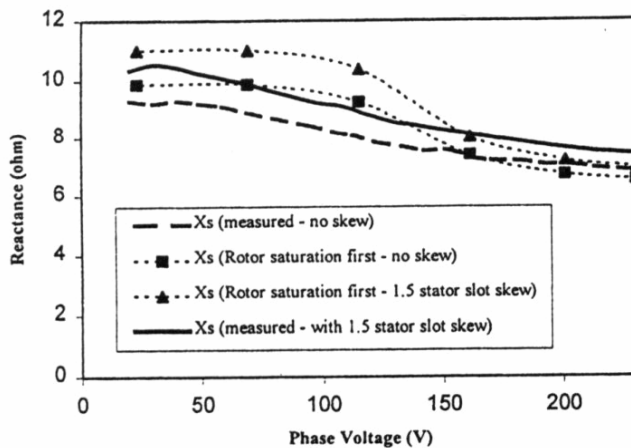
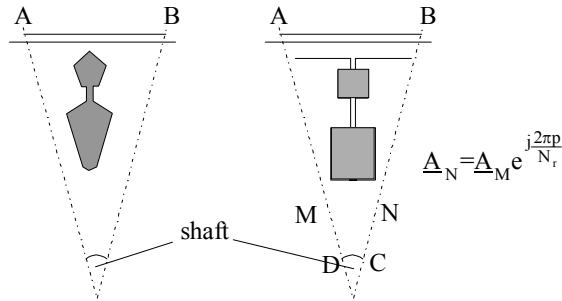


Figure 9.33 Leakage reactance for a skewed rotor versus voltage at stall

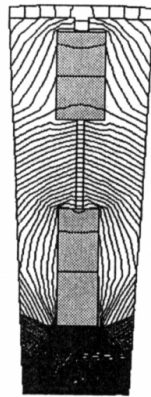
9.8. THE FEM APPROACH

The FEM has so far proved successful in predicting the main path flux saturation or the magnetization curve $L_m(I_m)$. It has also been used to calculate the general shape deep bar [17] and double cage bar [19] resistance and leakage inductance for given currents and slip frequency (Figure 9.34).

The essence of the procedure is to take into consideration only a slot sector and apply symmetry rules (Figure 9.34b).



a.)



b.)

Figure 9.34 a.) Deep bar and double-cage rotor geometry; b.) FEM flux line

The stator is considered slotless and an infinitely thin stator current mmf, placed along AB, produces the main flux while the stator core has infinite permeability. Moreover, when the stator (and rotor) mmf have time harmonics,

for inverter-fed situations, they may be included, but the saturation background is “provided” by the fundamental to simplify the computation process.

Still the zig-zag and skewing flux contributions to load saturation (in the teeth tips) are not accounted for.

The entire machine, with slotting on both sides of the airgap and the rotor divided into a few axial slices to account for skewing, may be approached by modified 2D-FEM. But the computation process has to be repeated until sufficient convergence in saturation then in stator and rotor resistances and leakage inductances is obtained for given voltage, frequency, and slip.

This enterprise requires a great amount of computation time, but it may be produced more quickly as PCs are getting stronger every day.

9.9. PERFORMANCE OF INDUCTION MOTORS WITH SKIN EFFECT

As already documented in previous paragraphs and in Chapter 7, the induction machine steady state performance is illustrated by torque, current, efficiency, and power factor versus slip for given voltage and frequency.

These characteristics may be influenced many ways. Among them the influences of magnetic saturation and skin effects are paramount.

It is the locked-rotor (starting) torque, breakdown torque, pull-up torque, starting current, rated slip, rated efficiency, and rated power factor that interest both manufacturers and users.

In an effort to put some order into this pursuit, NEMA has defined 5 designs for induction machines with cage rotors. They are distinguishable basically by the torque/speed curve (Figure 9.35).

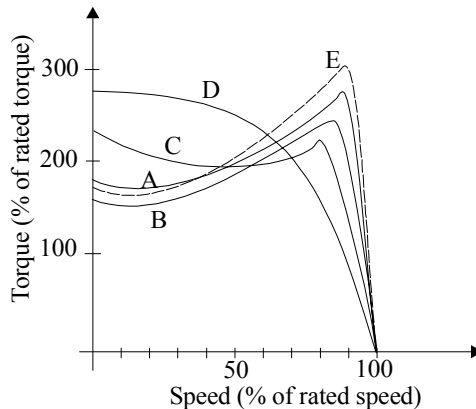


Figure 9.35 Torque/speed curve designs

Design A.

The starting in-rush current is larger than for general use design B motors. However, starting torque, pull-up and breakdown torques are larger than for design B.

While design A is allowed for larger starting current for larger starting torque, it also larger. These characteristics are obtained with a lower leakage inductance, mainly in the rotor and notable skin effect. Mildly deep rotor bars will do it in general.

Design B.

Design B motors are designed for given maximum lock – rotor current and minimum breakdown, locked-rotor and pull-up torques to make sure the typical load torque/speed curve is exceeded for all slips. They are called general purpose induction motors and their rated slip varies in general from 0.5 to 3 (maximum 5)%, depending on power and speed.

A normal cage with moderate skin effect is likely to produce design B characteristics.

Design C.

Design C motors exhibit a very large locked-rotor torque (larger than 200%, in general) at the expense of lower breakdown torque and larger rated slip—lower efficiency and power factor—than design A & B. Also lower starting currents are typical ($I_{\text{start}} < 550\%$). Applications with very high break-away torque such as conveyors are typical for Design C.

Rather deep bars (of rather complex shape) or double cages are required on the rotor to produce C designs.

Design D.

Design D induction motors are characterized by a high breakdown slip, and large rated slip at very high starting torque and lower starting current ($I_{\text{start}} < 450\%$).

As they are below the torque/speed curve D (Figure 9.35) it means that the motor will accelerate a given load torque/speed plant the fastest. Also the total energy losses during acceleration will be the lowest (see Chapter 8, the high rotor resistance case).

The heat to be evacuated over the load acceleration process will be less stringent. Solid rotors made of iron without or with axial slits or slots with copper bars are candidates for design D motors. They are designated to applications where frequent accelerations are more important than running at rated load. Punch presses are typical applications for D designs as well as very high speed applications.

Design E.

Design E induction motors are characterized as high efficiency motors and their efficiency is 1 to 4% higher than the B designs for the same power and speed. This superior performance is paid for by larger volume (and initial cost) and higher starting current (up to 30%, in general).

Lower current density, lower skin effect, and lower leakage inductances are typical for design E motors. It has been demonstrated that even a 1% increase in efficiency in a kW range motor has the payback in energy savings, in a rather loaded machine, of less than 3 years. As the machine life is in the range of more than 10 to 15 years, it pays off to invest more in design E (larger in volume) induction motor in many applications.

The increase in the starting current, however, will impose stronger local power grids which will tend to increase slightly the above-mentioned 3 year payback time for E designs.

9.10. SUMMARY

- As the slip (speed) varies for constant voltage and frequency, the IM parameters: magnetization inductance (L_m), leakage inductances (L_{sl} , L_{rl}'), and rotor resistance R_r' vary.
- In general, the magnetization flux decreases with slip to 55 to 65% of its full load value at stall.
- As the slip frequency (slip) increases, the rotor cage resistance and slot body leakage inductance vary due to skin effect: the first one increases and the second one decreases.
- Moreover at high slips (and currents), the leakage flux path saturate to produce a reduction of stator and rotor leakage inductance.
- In rotors with skewed slots, there is an uncompensated skewing rotor mmf which varies from zero in the axial center of the stack to maximum positive and negative values at stack axial ends. This mmf acts as a kind of independent magnetization current whose maximum is phase shifted by an angle slightly over 90° unless the rotor current is notably smaller than the rated value.
- At stall, the skewing magnetization current I_{mskew} may reach values 3 times the rated magnetization current of the motor in the axial zones close to the axial stack end.

As the main flux is smaller than usual at stall, the level of saturation seems to be decided mainly by the skewing magnetization current.

- By slicing the stack axially and using the machine magnetization curve $L_m(I_m)$, the value of $L_m(I_{mskew})$ in each segment is calculated. Severe saturation of the core occurs in the marginal axial segments. The skewing introduces an additional notable component in the rotor leakage inductance. It reduces slightly the magnetization inductance.
- By including the main flux, slot neck flux, zig-zag flux, differential leakage flux, and skewing flux per tooth in the stator and rotor, for each axial segment of stack, the stator and rotor saturated leakage inductances are calculated for given stator, rotor, and magnetization current and slip frequency.
- The skin effect is treated separately for stator windings with multiple conductors in slots and for rotor cages with one bar per slot. Standard correction coefficients for bar resistance and leakage inductance are derived. For the general shape rotor bar (including the double cage), the multiple layer approach is extended both for the bar and for the end ring.
- The end ring skin effect is notable, especially for $S \geq 1$ in applications such as freight elevators, etc.

- Skin and leakage saturation effects in the rotor cage are beneficial for increasing the starting torque with less starting current. However, the slot depth tends to be high which leads to a rather high rotor leakage inductance and thus a smaller breakdown torque and a larger rated slip. Consequently, the efficiency and power factor at full load are slightly lower.
- Skin effect is to be avoided in inverter-fed induction motors as it only increases the winding losses. However, it appears for current time harmonics anyway. Leakage saturation may occur only in servodrives with large transient (or short duration) torque requirements where the currents are large.
- In closed slot cage rotor motors, the rotor slot iron bridge saturates at values of rotor current notably less than the rated value.
- Skin and leakage saturation effects may not be neglected in line-start motors when assessing the starting, pull-up or even breakdown-torque. The larger the motor power, the more severe is this phenomenon.
- Skin and leakage saturation effects may be controlled to advantage by slot geometry optimization. This way, the 5 NEMA designs (A, B, C, D, E) have been born.
- While skin and leakage (on load) saturation effects can be treated with modified 2D and 3D FEMs [19, 21], the computation time is still prohibitive for routine calculations. Rather sophisticated analytical tools should be used first with FEM and then applied later for final refinements.

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